Coherent discourse and early number teaching

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Abstract

Functional linguists argue that a prerequisite for the mediation of semiotic meaning is coherent discourse. The multisemiotic nature of mathematics adds complexity to the need for coherence. In mathematics classrooms it is language that weaves semiotic modes together as teachers' words and explanations are the means by which the relation within, between and across semiotic modes are explicated. The assumption is that there are greater possibilities for mediating semiotic meaning when there is coherence in the teacher talk and practice that seeks to convey such meaning. The focus of this paper is to illustrate, through comparative analysis of discourse in two Grade 2 number lessons – one, an intervention project number activity and the other a number lesson taught by a Grade 2 teacher - the significance of coherent discourse for enabling sense-making of the number concepts taught.

What teachers and learners say, do, and write are the experiences that form the bases for meaningful learning. Coherence or incoherence across these semiotic modes create or militate against possibilities for meaning making. The concept of coherent discourse, drawn from systemic functional linguistics and operationalised in relation to literature of early number learning, provided the lens to analyse the pedagogic discourse in the two classrooms. The paper firstly identifies functional/qualitative differences in coherent discourse in two lessons and secondly, identifies reasons for lack of coherence. These reasons enable or militate against meaning making resulting in differentiated possibilities for development of learners' number sense. The implications of the differing discourses for the mediation of semiotic meaning and for teacher development are discussed.

Introduction

Functional linguists argue that a prerequisite for the mediation of semiotic meaning is coherent discourse (Halliday and Hasan, 1991). The assumption is that there are greater possibilities for semiotic meaning making by learners when there is coherence in the discourse that seeks to convey such meaning Hasan (2004), drawing on Vygotsky (1978), defines the concept of semiotic mediation as "the cultural mediation of mental development through acts of semiosis" (p.30). Semiotic acts are 'acts of meaning' mediated by signsystems such as language. While semiotic mediation of meaning is a prerequisite for the development of higher mental functions, a prerequisite for semiotic mediation is coherent discourse (Hasan, 2004). Discourses are not just linguistic descriptions and thoughts but include the practices that arise from them. Thus, "discourses are practices that systematically form the objects of which they speak" (Foucault, 1977, p.2), and are inclusive of accompanying language, visual representations and practices. From the perspective of numeracy teaching, the focus on discourse therefore includes what teachers say, do and write on the board, and their responses to learners, as the experiences for learners that form the bases for meaningful learning.

The fundamental role that discourse plays in any classroom has been highlighted by Wertsch and Minick (1990). They argue that a fundamental property of discourse in classrooms is that the reality it creates is text-based. The term 'text-based reality' indicates that a discursive reality is created by semiotic means: "A variety of kinds of texts and realities may be involved, but in all cases a reality or 'problem-space' is created, maintained and operated on through semiotic (usually linguistic) means" (Wertsch and Minick, 1990 p.74). Similarly Halliday and Hasan (1991, p.95) note that while other semiotic modalities such as eye-contact, gestures and facial expressions are used to mediate meaning "the meanings relevant to a topic must be created through appropriate, communally interpretable language".

O'Halloran (2000) emphasises the multisemiotic nature of mathematics – with modes ranging across symbolism, visual display and language. Important in the context of early learning, Schleppegrell (2007) distinguishes between oral language and written language, and adds a focus on gestures and actions. Haylock and Cockburn (2008) describe early number learning in terms of making strong connections between actions on objects, the words used to describe these actions, diagrams that represent essential components of these actions, and the mathematical symbols that can concisely and conventionally represent these actions/words. Schlepegerell (2007, p.142) also highlights the need for coherence across semiotic modes:

> the written language, the mathematics symbolic statements, the visual representation and the oral language **work together** to construct meaning as the teacher and students interact in (Schlepegerell, 2007, p.142).

Veel (1999), discussing pedagogy in the context of linking different representations, notes that: "teachers words and explanations are needed to interpret the meanings that the visual displays and symbolic representations construct," (Veel, 1999, p.189) as the teachers words are the means by which the relation between representations is conveyed. Thus, while the use of concrete aids, visual representations and gesture/movement may be used to illustrate a number concept, the building of progressive sense relations is usually achieved through linguistic extensions and elaborations of meaning.

This analysis of pedagogic discourse draws on the concept of coherent discourse. Systemic functional linguistics (SFL) provides analytical distinctions of coherent discourse that enable its description. Discourses that have the property of coherence are discourses that are strongly connected through the use of structural and textual devices. Whether a discourse is coherent and therefore, creates possibility for meaning making or impedes it, is empirically analysable. The focus of this paper is to illustrate, through analysis of two Grade 2 number lessons, differences in coherence and differentiated possibility for semiotic mediation that can be seen through application and elaboration of the notions of structural and textual coherence.

In the next section, Halliday and Hasan's (1991) SFL concept of coherent discourse has been drawn on together with O'Halloran's emphasis on multisemiotic modes in mathematics. Whilst O'Halloran's focus has been on secondary mathematics ideas, our focus is on early number learning where the need to connect learners' everyday understandings of number with the extensions that become possible and necessary through work with visual and symbolic representations of number, are important. We use the idea of coherence through such connections to compare two sections of pedagogy in order to understand the nature and extent of connections made in the two texts. Given the emphasis on connections as central to conceptual understanding and meaning making in mathematics (Hiebert and Lefevre, 1986), this analysis allows us to understand potential differences in possibilities for learning, and as such, ways of furthering our understandings for teacher development for numeracy.

Conceptualising coherent discourse for number learning

Halliday and Hasan (1991) identify features of coherent discourse that are pertinent for our analysis. For them, a text that is characterised by coherence hangs together: “At every point after the beginning, what has gone before provides the environment for what is coming next. This sets up internal expectations. . .” (p 48). In this view, prior discourse forms an important context for making sense of subsequent discourse. An important contribution to coherence comes from cohesion: “the set of linguistic resources that every language has for linking one part of the text to another” (Halliday and Hasan 1991, p.48). For a text to achieve internal cohesion it has to have the property of unity. Unity in written and spoken texts is of two types – unity of structure and unity of texture. Halliday and Hasan (1991) argue that a text with unity of structure is made up of separate events or elements that are connected. Three separate events are identified – the beginning or the precipitative event that propels from one stage to another; the consequential event that arises from the precipitative event; and the revelatory event that leads to redefinition of the precipitative event. This Aristotelian definition of structure is therefore based on the three elements: a discernible beginning, middle and end. The pedagogic discourse of teachers across the two focal lessons was firstly analysed for structural continuity.

Textual continuity refers to meaning relations within phrases and utterances and between phrases or sentences so that the meaning of the larger piece of language is achieved by the links between the smaller units. The linguistic concept of cohesive tie (Halliday and Hasan, 1991) focuses attention on meaningful ties within, between, and across text. Coherent explanations are characterised by strong cohesive ties across individual messages of a text that produce continuity in the discourse. Within numeracy teaching, and following O’Halloran (2000), messages can be communicated and linked through what the teacher says, writes (using words and diagrams), does and learner responses. For example the drawing of a number line on the board, accompanied by the words, ‘this is a number line’, and the writing of the term number line on the board indicates strong cohesion across what the teacher says and does and writes that increases coherence across these activities and therefore possibilities for appropriating the meaning of ‘what a number line is’.

Textual continuity is achieved through three types of cohesive ties – co-referentiality, co-classification and co-extension (Halliday and Hasan, 1991). Within each of these types, we link the descriptions provided in Halliday and Hasan’s (1991) work with examples drawn from the terrain of early number learning.

Co-referentiality can refer to the use of pronominals, such as ‘he, she, it’ and the use of the definite article ‘the, this, that’ with reference to the subject of the previous sentence. For example, in the sentence ‘Right, now,. . . the summary there. . . it says. . .’, the use of the pronominal ‘it’ refers to ‘the summary’ unambiguously. Co-references enable efficient use of language as the subject of the sentence or the previous sentence need not be repeated. Co-references used ambiguously can introduce incoherence in a text especially if there are two subjects that could be referred to or, if across a few sentences, different subjects could be referred to. Ambiguity refers to the use of a cohesive device such as a co-referential in a way that allows more than one interpretation or meaning to be attached to it. An example of co-reference in the context of early number learning would be: ‘14 is an even number, so we can share *it* equally between two people’.

Co-classification can refer to the use of substitution or ellipsis in a text. In substitution the second message further classifies the first without repeating it. For example, in the two sentences ‘Right, now, the test on Wednesday. . . You **need to know** everything up to and including what we have done today’ indicates the use of substitution. The second sentence refers to ‘the test’ that was stated only in the first sentence. In ellipsis the second sentence is meaningful only in relation to the first and a distinct case of it. Co-classifications used ambiguously will also introduce incoherence in a text. Within early number learning, and in mathematics more generally, an alternative representation of an idea can be thought of in co-classification terms – e.g. ‘I am going to draw a number line’ introduces a diagrammatic representation that unambiguously connects to the words ‘number line’. In mathematics, equivalent representations across multisemiotic modes provide alternate ways of seeing an idea that stress particular features of the idea whilst backgrounding others (Mason and Johnston-Wilder, 2004).

Co-extensions are content words or lexical items in a field of meaning. Co-extensions are produced by three types of meaning relations – the use of synonyms, the use of antonyms and the use of hyponyms. Synonymy is the use of words that are similar in meaning to the key term that evoke identical experiential meaning, e.g. the use of ‘take away, minus, less than’ to convey the concept of ‘subtract’ and the use of the word ‘middle number’ to enable learning of the concept of half. Antonymy refers to the use of words that mean the opposite that also evoke experiential meaning by saying what it is opposite of. For example the teacher might use the words ‘not before’ to convey the meaning of ‘after’.

Hyponymy refers to explaining a concept by classifying it in terms of its general class and its sub-classes: the focus is on general-specific relations. For example, the concept of half is general and decontextualised, whilst ‘5 bricks is half of 10 bricks’ is a specific instantiation of it. Generality moves in stages in early number learning – Hughes (1986) notes the more abstract nature of ‘What is 2 + 2’ in abstract number terms in comparison to ‘What is 2 bricks and 2 bricks in all’? Mason and Johnston-Wilder (2004) also note the central role of ‘specialisation’ and ‘generalisation’ of examples, and the classes they are exemplars of in mathematical learning more broadly. General or abstract sense-relations are difficult for children to grasp but are powerful precisely because their sense is not dependent on specific contexts. A key aim of numeracy in the Foundation Phase is to support the development of the gradually more general and abstract sense-relations of number that are needed for progress in Intermediate Phase mathematics (Department of Basic Education (DBE), 2011a, 2011b).

An additional sense relation, i.e. meronymy (Halliday and Hasan, 1991), refers to compositional relations (or part of) where the focus is on part-whole relations, for e.g. a tree and its parts such as roots and branches. In relation to developing understandings of the meaning of 16, explanation of number bonds, e.g. 9 + 7 = 16, would represent a meronymic co-extensional sense relation. In mathematics, ‘elaboration of a concept’ can be considered in terms of co-extensions of meaning. For example, ‘decomposing 16 into two constituent parts’ is a general concept, which has several specific instantiations, 9 + 7, 10 + 6, etc. In mathematics, movement between general-specific or part-whole relations can work in both directions with an emphasis on deducing specific instantiations, using the known whole or part to work out the unknown, and focusing on working systematically to generate all cases that fit a given constraint.

Hasan (1991) further notes the role of repetition in achieving coherence in texts because ‘the repetition of the same lexical unit creates a relation simply because a largely similar experiential meaning is encoded in each’ repetition.

In order to employ the above concepts to analyse the data we needed to operationalise them into indicators of more and less coherent discourse. This operationalisation, which brings the concepts in conversation with aspects of early number learning that were pertinent to our dataset, is reflected in Table 1 below:

Table 1: Operationalisation of the key concepts

Coherence	More coherent discourse	Less coherent discourse
Unity of structure	A text that has a clear and connected beginning, middle and end. A text in which the separate parts are connected. 1. Within a part in the text there is connection across problem space, consequential event and redefinition	A text that has unmarked or vague beginning, middle and end. A text that has separate events that are unconnected. Parts in the text lack connection between problem space, consequential event and redefinition.
Unity of texture	Unambiguous use of co-references – e.g. it. The co-referents can only be interpreted in one way.	Ambiguous use of co-references. The co-referents could be interpreted in more than one way.
	Unambiguous use of co-classification. The substitution/ellipsis is clearly indicated across the sentences.	Ambiguous use of co-classification – substitution and ellipsis. The learner could substitute different meaning than intended.
	Synonyms used to mediate meaning. Synonyms used for key terms such as increasing/more, decreasing/ less, doubling/ halving/middle in our data.	Lack of use of synonyms to convey similar experiential meaning. The word to be mediated is repeated, rather than elaborated through other words that have similar meaning.
	Antonyms used to mediate meaning of key terms – left/ right, first/last, before/after, plus/minus, more/less, addition/subtraction,	Lack of use of antonyms to convey meaning of word. The word to be mediated is repeated and not elaborated through its opposite meaning, e.g. the number after, not before 6 is 7.
	Orders super-ordinate/hyponymic or general-specific relations that shows hierarchy and connectivity of concepts.	Lack of ordering of super-ordinate/ hyponymic relations or general-specific relations. Meanings are either contextualised and specific or general and abstract.
	Part-whole relations are clear. Partitioning of numbers that make the part/whole clear.	Lack of part-whole relations. The part is disconnected from the whole or the whole is not understood in terms of its parts.
Repetition	Effective repetition of key phrases, combined with textual coherence devices that create the same experiential meaning in different parts of the text.	Ineffective repetition. Repetitions of phrases such as ‘repeated addition’ that fail to create experiential meaning for learners.

Research design

Within the context of poor numeracy performance in primary schools (Department of Education (DoE), 2008) work began on a longitudinal research and development project – the Wits Maths Connect – Primary project (WMC–P) – focused on developing and investigating the implementation of interventions focused on improving the teaching and learning of primary mathematics in ten government primary schools. As part of the baseline data collected for this project, the project team observed and videotaped a single numeracy lesson across the Grade 2 classes in the ten project schools, with a view to gaining insights about the nature of teaching and learning, and the classroom contexts of these activities.

Initial post-observation discussions in the project team involved comments about a lack of purpose and connection between ideas in lessons. Disconnections between a range of aspects of classroom talk and activity were noted – the object of learning/teacher explanations and teacher explanations/materials being used to support the activity. Later in the year, as part of the project's intervention work, short activities based on building early number sense were taught by project team members. In this paper, the significance of coherence for early number teaching is exemplified through comparing the nature and extent of coherence between one of these number sense activities and a Grade 2 number lesson observed early in the year. The first lesson is a number sense activity taught by the project leader and the other is a number lesson taught by a Grade 2 teacher in a suburban school. Data for the number sense activity was collected through non-participant observation and writing detailed field notes as the activity progressed. The field notes were later reconstructed, filled in and typed. The Grade 2 teacher's lesson was observed and video recorded. The video records were then transcribed into text that included what the teacher said, wrote on the board, her actions and her learners' responses. The transcripts were then divided into episodes – with new episodes signalled by the introduction of a new task.

The number sense activity was made up of 11 episodes: (1) introduction to number line; (2) positioning18 on a 1-20 number line; (3) positioning 7 on the number line; (4) identifying middle of line; (5) identifying what number is in the middle; (6) positioning number 18 in relation to middle number; (7) positioning 7 in relation to middle number and connecting this to associating 'less than' and 'before'; (8) repeating above with 12 on the number line; (9)

finding middle of 10 and 20 on number line; (10) using 10-20 mid-value to re-look at position of 12; and (11) an individual assessment activity looking at number recognition, positioning and ordering on a 1-12 number line.

The Grade 2 teacher's lesson was made up of 12 episodes: (1) forward counting from 1–100, then backwards from 100-1; (2) an addition word problem with answer 16; (3) writing 16 in numerals; (4) drawing 16 objects; (5) counting 16 objects/counters; (6) representing 16 as a number word; (7) representing 16 in pictures; (8) recognition of 16 as a number and a word; (9) number pairs that add to 16; (10) subtracting two numbers to make 16; (11) repeated addition of a number to make 16; and (12) a written exercise with addition and subtraction sums to make 16. The teacher's stated aim for the lesson was: 'There are many ways to make 16 – different kinds of methods can be used to make the number 16. The important thing here is for you to know how to write 16 in number, 16 in words – the number name – and how many pictures are we talking about when we talk about the number 16'.

The data transcripts were analysed according to the analytical framework developed reflexively from both the SFL concepts described in Table 1, with attention to coherence between multisemiotic modes. The units of analysis were the teachers' utterances – their words, sentences and explanation sequences, incorporating what they wrote on the board and how they responded to learner inputs. The first step was to analyse the overall structural unity of the lesson and then structural unity across the various parts of the text. The second step was analysing textual unity in terms of sense relations within and across the utterances. Lastly, the use of repetition in the lesson was analysed. (See Appendix A for annotated sections of our analysis using this approach for both lessons). For coherence to be established, structural and textual coherence are simultaneously necessary – they have been separated here to facilitate analysis.

The presence of multisemiotic modes in both lessons

Both lessons evidenced mutlitsemiotic modes: oral language, written language, visual displays and gestures or actions (manipulation of) using mathematical aids such as abacus in varying degrees. Key differences were the extent to which written and oral language accompanied and connected the visual displays, symbolic and concrete representations using counters and abacus. Variations also arose from varying structural and textual continuity/connection within and across the semiotic modes. Variations were also noted in the use of repetition.

Structural unity of the lessons

In the number sense activity, the beginning consisted of drawing a number line on the board and indicating 0 and 20 on it followed by assessing learners' ability to recognise 0, 20 and 18. The middle was a sustained focus on positioning various numbers on the 0–20 number line using a range of concepts to justify the position: half-way numbers, less than, more than, before and after. Co-extensions such as halfway, middle, same number on either side, bigger than and smaller than – were introduced and linked to the number line. Several examples were worked through with the whole class on estimating where numbers could be placed on the number line by halving numbers through equalising the two parts created. The lesson ended with an application exercise where learners were asked to draw a number line and indicate the position of numbers given to them on it.

Analysis of the Grade 2 lesson transcript indicated that the early episodes consisted predominantly of counting and matching activities – all involving number recognition and matching activities across word, numeral and pictorial representations. The middle section (parts 9–11) was constituted by 3 exercises involving addition, subtraction and repeated addition operations to produce 16. In each of these parts several examples were completed by the teacher with the whole class. The shift to a new operation was signalled by the teacher but not connected to the previous operation. The lesson ended with an application exercise where learners were given a set of problems (addition and subtraction) that equaled 16.

While both lessons showed a clear beginning, middle and end they differed in terms of unity across parts of the lesson. In the number sense activity each part after the beginning was linked to the previous part and formed the context for the next part. The Grade 2 lesson illustrated weaker structural unity due to the lack of explicit connection across the 12 elements. Connections were not made across sums within episodes, and between the addition, subtraction and repeated addition episodes. Consequential steps therefore tended in almost all instances, to de-link, rather than connect with prior solutions.

Textual unity of the lessons

In this part of the analysis the coherence in meaning relations within phrases and sentences and across phrases and utterances and with accompanying activities in both lessons are analysed.

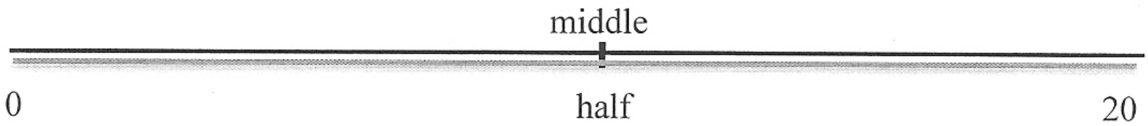
Number sense activity

In the number sense activity, co-referentials were used unambiguously throughout. For example ‘this’ was used four times in episode 1. The first ‘this’ referred to the number line, the second ‘this’ referred to the number 0, the third referred to the number 20 and the fourth to the number 18 – with each denoted by pointing to the referent. When the co-referential ‘it’ was used, its meaning was unambiguous: ‘If I wanted to put 18 on the number line, where would I put it?’ – ‘it’ referred unambiguously to the number 18 which was held up on a card and written on the board. The practice of writing each aspect on the board provided linkages across spoken language, symbols and diagrammatic representations in the public classroom space – a feature that was absent in several episodes of the Grade 2 teacher’s lesson. Part 2 of the activity focused on where 7 should be on the number line, justifying its positioning, and connecting 7 being smaller than 18 to its positioning before 18.

Co-classificational elaboration of meanings related to the position and relative size of number could be linked to the numbers selected being represented on the number line. This representation links quantity to position –and thus, the co-classification opens space for co-extensions that push towards more abstract representations of number – a feature that has been noted as important if children are to get to grips with number in the concept terms needed for subsequent mathematical learning (Gray, 2008). The number line drawn on the board provided a co-classificational equivalent elaboration – a representation of the numbers that incorporated relative position as a feature (Haylock, 2006). Pointing out 0 and 20 and then holding up 18 and asking ‘what number is this?’ and getting the answers from learners provided a check of co-classificatory ability between number name and symbol recognition of 0, 20 and 18. Decisions on positioning were also linked to co-extensional elaboration based on synonyms – middle, half way, equal lengths on both sides, as well as ‘smaller’ to ‘before’ within the number line representation. Whilst there are mathematical features of the number line model that are not dealt with in the teacher talk – the continuous nature of number needed to make sense of the measurement notion contrasted with the discrete nature of the integers being dealt with here, the selection of the number line model allows for the measurement idea implicit in the idea of 'halving' to come into view, and linked to the positioning of the counting numbers that learners are already familiar with. The visual mediator thus lays the ground for potential expansions of number concepts that are likely to be useful further down the line.

There was also cohesion between the task: 'Who can tell me where 7 is on the number line?' and the response from the learner who pointed out the correct position of 7. Further, the opportunity for all learners to see the response and hear the feedback given to the learner enabled possibilities for making individual meaning making more visible to the whole class.

Part 3 illustrated the use of synonyms to mediate meaning in the provision of justifications for the particular placing of 7 on the number line. To do this, the middle of the number line was established and 'middle' was written above the mark and 'half' was written below the mark, as below:



Part 3 moved learners into thinking more precisely about the middle of this number line. The mid point was agreed with learners and a mark was made on the line to show the mid point. In part 3 co-extensions for 'middle' such as 'same distance' and 'same gap' were used to lead to the concept of half. In addition the same distance on either side of the middle point was shown by gesturing the same width from 0 to the middle point as from the middle to the 20. Again the use of the co-referential 'this' three times was unambiguous as it was accompanied by pointing out what was being referred to on the number line.

Part 4 went back to the key question 'so what number is the middle'. The incorrect response of '8' from a learner was probed further, and shown to be incorrect with the correct response of 10 checked and written on the number line.

Part 5 illustrated a hyponymic relation. Following an elaborated co-extension of 18 in terms of both symbolic representation and order, the notion of 'less than' was linked to the positional relation 'before' on the number line. The number 18 provided the specific example, but the rule was stated as a general principle: 'When a number is less than – it comes before.' Then, since it was just 2 away, it was placed at the end of the line, closer to 20. The co-extensional technique of hyponymy was evident in that the general principle – each number on the number line is bigger than – more than the ones before it and smaller than – less than the ones after it – was mediated through a number of examples to illustrate the principle.

Antonymy was used often as well – 'bigger than' and 'smaller than', 'less than' and 'more than', 'before' and 'after' to enable meaning mediation. The excerpt below illustrates the use of 'more' or 'less than' and 'less or more than'.

H: Is 18 *more or less than* 20?

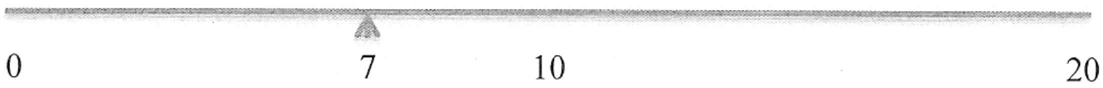
L: Less than 20

H: Writes on the board – 18 is less than 20

H: The other number is 7, where should 7 be?

H: Is 7 *less or more* than 10, asks L to point out where 7 should be. . . [Lr comes up to indicate position]

H: Writes 7 and arrow to point to 7 on the number line.



Analysis of the number sense activity indicated that repetition was linked to the incorporation of synonyms: the key synonym 'middle' was repeated 15 times in different examples and the term 'halfway' was repeated 16 times in different examples to enable understanding of the concept of half on a number line. Linked also with gestural actions of 'travelling' the same distance on either side, there were therefore, multiple processes supporting possibility of learning the general meaning of 'half' of a given number.

In sum, multiple connections, ranging across the different types of cohesive types detailed in Halliday and Hasan's theory, and ranging across the multiple representational modes that O'Halloran (2000) suggests are a feature of mathematical working, are seen within this activity. These connections in turn, point to strong textual continuity.

The Grade 2 'baseline' lesson

This lesson has been analysed in detail in a previous paper (Venkat and Naidoo, 2012). This previous analysis is drawn on and elaborated further here to enable comprehension of the specific variations in coherence. Within the shorter earlier episodes (1–8), there were relatively strong cohesive ties across what the teacher said, wrote on the board and what learners were asked to do. Of interest is the fact that some learner errors were missed, and that co-reference and co-classification, whilst used coherently, tended to be used for teaching equivalence across numeral, word, diagram and counting activities – which for the number 16 would fall within Grade 1 rather than Grade 2 curriculum content (DoE, 2008). Also of interest in relation to the counting activities, was the fact that concrete unit counting was promoted across all episodes with no scaffolding into what Ensor, Hoadley, Jacklin, Kühne, Schmitt and Lombard *et al*. (2009) refer to as a more abstract calculating orientation. Given that particular problems were evident in learner responses in Episodes 9, 10 and 11, finding two numbers that add to 16, subtraction of numbers to give 16 and repeated addition to give 16 respectively, we focus on these three episodes, whilst making reference to features drawn from the other episodes.

A key general feature of the lesson was the lack of sharing of a representation in the private space of the learner – e.g the child who counted out 16 counters on the floor in Episode 5, and teacher talk on the learner's representation in the public space of the class. Here, learners could hear the teacher but could not see the representation being referred to – and this recurred in other instances as well. Halliday andHasan (1991) notes the importance of connection across oral and graphic discourse, and also the distinctions – in particular the relative permanence of graphic representations in relation to the ephemerality of talk. Leaving out a key step of showing the individual learner's representation to the whole class, either on an abacus or through drawing on the board excluded the majority of learners from accessing explanations given by the teacher that were scaffolded with co-classificatory representations.

The teacher introduced part 6 with ‘now boys and girls I want you to give me 2 numbers, when we add them together they give us number 16’. This instruction was repeated four times until a learner gave the answer of 8 + 8, and followed by asking a learner to check whether 8 + 8 made 16 by counting out 8 and 8 and adding. She then asked for another two numbers…another learner offered ‘9 + 9’, which learners were asked to check on their abacus. Some learners appeared unable to count out 9 and 9. In this part there is a break in communication as many learners were unable to make an accurate representation and were not given instructions that helped with this. In terms of pedagogy the fact that 16 is the ‘given’ here, and that the task requires the generation of various partitions of 16 was not communicated.

Analysis indicated repetition here without explicit focus on the task constraints, and limited co-extensional elaboration of meaning. When learners called out pairs of numbers, each was checked by counting on the abacus and then the correct sums were written on the board. In part 10 the teacher shifted to subtraction to make 16. A similar procedure was followed with learners being asked to give two numbers that when subtracted gave 16. The lack of co-classificational or co-extensional elaboration here keeps the activity of generating two numbers that add to 16 in the realms of concrete trial and error, followed by empirical verification, rather than more cohesively supporting the move to a deductive strategy through which appropriate partitions can be derived, rather than guessed.

Synonyms such as add and plus and take away, minus and subtract were used in parts 6 and 7. We note that in both instances, the terms offered are relatively ‘formal’ mathematical terms, and that the actions on concrete objects remained in the terrain of individual learners working on their abaci. Thus, co-classificational connections tended to remain once again, in private, rather than the public classroom space.

Mediation between representations in public and private spaces was also problematic in the repeated addition episode. After repeated instructions a learner offered eight 2s (which she had made on her abacus). The teacher acknowledged her answer and re-explained to the whole class that this learner had got ‘2 eight times’ and this had given her 16, referring to the girl’s abacus, but did not show the whole class the arrangement. The teacher then wrote 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16 on the board – a shift to a symbolic form of co-classification. Learners were then asked to make the 8 groups of 2s on their abacus, but several learners were not able to do the task – one learner pulled down ten 2s on her abacus; another has 10, 7 and 1 pulled down; another five 3s and a single bead pulled down. What was evident was their inability to translate the instruction into appropriate concrete representations on their abacuses. Here, co-classification was presented, but not in ways that connected with learner competences. The lack of visibility of the concrete representation seemed to contribute to learners' inability to link the co-classificational form with its concrete equivalent. Thus, the two semiotic modes were not integrated into a single semiotic system.

Instances of ambiguous use of co-references have been noted in Venkat and Naidoo (2012) and are drawn on here for purposes of comparison. For example, in the introduction to part 8, the teacher introduces the focus on repeated addition with the following words:

> 'Now, who can tell me, you look for one number, you look for one number, you add it many times, it gives us 16. Only one number, you add it many times, repeated addition, to give us the number 16. You must work it out on your abacus, stop colouring. Work it out on your abacus. Only one number.'

The ambiguity arises in the frequent use of 'it'. In this excerpt, the repeated use of the pronominal 'it' has been underlined. The first 'it' refers unambiguously back to the 'one number' that is being looked for which can be added many times to give 16. The second 'it' seems to refer to the output of the process of repeated addition rather than the 'one number' that is being added. The lack of explicitness of the shift of reference from the number that is being added repeatedly to the output of the process of repeated addition creates ambiguity, and particularly so for a learner still grappling with this process. This ambiguity is compounded by the fact that the third 'it' refers again to the starting number that is being added repetitively. The fourth and fifth uses of the word 'it' seem to refer to the arrangements that have to be produced to make 16 to repeated addition – essentially 'it' here refers to the abacus arrangements that the teacher wishes learners to produce. Overall therefore, the number of shifts of reference within this short introduction is likely to contribute to ambiguity for a learner trying to comprehend the meaning of the 'it' in consecutive sentences.

Repetition of procedural instructions in the teacher's lesson was not linked to elaboration of the meaning of the instruction, e.g. see the italicised phrases in the excerpt below from episode 9:

> Now boys and girls, *I want you to give me – two numbers, when we add them together, they give us number 16. Two numbers, when we add them together, they give us number 16.* [Some hands go up immediately.] Have you done it first? How do you know it is 16? You have to work it out first. *The two numbers, when we add them together, they give us 16. And – don't – make – noise.* When you are counting, make sure that you don't make noise. *Two numbers, when we add them together, they give us number 16.* Right? What?

In the above excerpt the instruction 'I want you to give me two numbers, when we add them together, they give us number 16' is repeated four times and in the entire episode on addition the same instruction was repeated 7 times.

In part 11 also, repetition was again present without co-classificational or co-extensional elaboration, and here, occurred with the ambiguity in co-reference presented earlier. In the activity seen in this episode, several learners appeared unable to generate an appropriate representation of repeated addition to make 16 on their abacuses. Weak cohesion therefore appears to relate to lack of use of co-extensions flexibly to mediate the meaning of repeated addition. This weak cohesion was in some instances compounded by lack of appropriate boundary setting in relation to the concept being taught:

> T: 16? I said – 16 – how many times did you add 16 to get 16? Sorry? You put 1 to 16? And it gives you 16? Ok, but that's not what I want. I said, one number, you add it several times. One number, you add it several times, and you tell me how many times did you add that number to give you the number 16. That is repeated addition.

Here, one group of 16 is not viewed as part of the set of appropriate responses for repeated addition to make 16. But repeated addition as a process in mathematics is an important part of the trajectory that leads to multiplication and factor pairs, and (1,16) can be viewed as an important example to include and discuss given this 'horizon' (Ball and Bass, 2009). In Halliday and Hasan's (1991) terms, problematic 'internal expectations' are established.

There were many more instances in this lesson of ambiguity within co-reference, and repetition without co-extensional elaboration. We therefore suggest that repetitions of key terms and phrases appear to contribute to coherence when linked with the other textual coherence categories that allow for elaboration and connection of meanings. A further point to note is the lack of connectivity across parts 9, 10 and 11 to mediate the concept of repeated addition. While the teacher repeatedly told learners what repeated addition was she did not direct the attention of learners to the relationships between examples within episodes or between addition and repeated addition. In the addition sums the pairs of numbers varied, whereas for 2+2+2+2+2+2+2+2 =16 the number 2 is repeated 8 times, for 4+4+4+4 =16 the number 4 is repeated. This lack of cohesion across parts 9, 10 and 11 militated against semiotic mediation of the concept of repeated addition.

Discussion

The analysis of discourse of the lessons show marked differences in coherence. The number sense activity illustrated greater structural continuity – it had a highly interlinked problem space that was connected across episodes and concluded with an exercise based on the goal of the lesson. The activity made use of mutisemiotic modes and maintained coherence across them. With reference to textual coherence there was greater connectivity across sentences – from one sentence to the next, across consecutive sentences, between what was said and written on the board so that it could be seen by all, from what was a correct response and announced to the whole class and written to be seen by all; between verbal and symbolic/ diagrammatic representation and between what learners were asked to do and what they did. The request for justification of answers across correct and incorrect responses further cemented the expectation that learners were to use the language and ideas seen in the problem space to elaborate their contributions. Secondly co-references were used less and were used unambiguously. Thirdly antonyms and synonyms were used to enable semiotic meaning making. Repetitions of key concepts were repeated with elaboration.

Whilst Halliday and Hasan (1991) holds that conversational texts can often withstand some incoherence and still maintain overall coherence due to the frequent presence of broader shared contextual understandings, pedagogic texts have much greater need for coherence. In the pedagogic arena of the Grade 2 baseline lesson, the lack of systematic recording on the board appeared to limit openings for the learners to see structural relationships between specific cases – which in turn blocked openings for concept building. The lack of structural continuity across the substantive parts of addition, subtraction and repeated addition negated openings for the preceding operation to provide opportunity to understand the relationships across the operations. Whilst learners were able to respond with examples, verification of their ‘correctness’ always occurred empirically, by making and counting. In this approach, the unit counting seen in Ensor *et al.*'s (2009) analysis of Foundation Phase teaching and in Schollar's (2008) learners' work is evident. A consequence of this approach is the complete de-linking of subsequent and prior examples, and a lack of 'building' unknown knowledge from known information – a feature that is viewed as central to building number sense (Anghileri, 2006) and mathematical learning more generally (Askew and Wiliam, 1995).

The lack of structural cohesion across addition, subtraction and repeated addition has implications for learning number sense. In instances where examples of sums making 16 were offered, these were generated from first principles, rather than 'derived' from the previous part of the lesson. It is this boundedness of parts 9, 10 and 11 – the localisation of working that in essence produced the sense of disconnection between and within episodes in the lesson. Given that literature in the field of early number learning within mathematics education has noted the centrality of developing the ability to generalise patterns and processes and link new problems with the knowledge they already have (Anghileri, 2006), this localisation of working to each immediate task within an episode is problematic. In particular, the ways in which teacher talk structures tasks within this lesson promotes a message of 'extreme localisation' (Venkat and Naidoo, 2012), which stands diametrically opposed to the need to encourage connections and cumulative learning.

Furthermore the lack of semiotic flexibility was evident in firstly, co-references used ambiguously, secondly, key terms/concepts not being co-extended sufficiently using sense relations of antonymy, synonymy and hyponymy and thirdly, repetition of instructions in the same words, rather than with appropriate co-classifications and co-extensions. The result is a repetitive reliance on trial-based guessing and checking sums concretely using the abacus, rather than being able to use deductive thinking (which in mathematics, would rely by definition, on connection with prior results). Thus, the means by which the 'holding back' in concrete methods that has been identified in prior findings (Ensor *et al*., 2009) – is seen here through a cohesion lens.

Implications for teacher development

Our analysis suggests the need for two linked avenues within our teacher development work. Firstly, at the technical pedagogic level, the need for systematic writing on the board and of showing the formations of individual learners to the whole class in ways that provide co-classificatory supporting representations of talk – would seem to be important. The use of strong cohesive ties across what is said, what is written or represented symbolically – in numbers and diagrams - on the board, and what is done. The need to balance individual and group instruction with whole class teacher led instruction to establish an ‘appropriate, communally interpretable’ discursive practice.

At the conceptual pedagogical level, the need to build elaborations of key mathematical ideas through language, also comes through, as a way of moving past the repetition that fails to provide learners with alternative routes to understanding the idea in focus. Within this focus on language, we note too, the need to understand the progression of early number ideas from concrete counting to more abstract number concepts that can only be promoted through coherence across multiple semiotic modes including co-classificatory and co-extensional elaborations. Overall, this suggests the development of metalinguistic awareness amongst teachers of the use of co-references, co-extensions and the use of effective repetition for the mediation of meaning within and across sections of texts and multisemiotic representations in progressive ways. The conceptual level may well be more complex to address, but without this, what we see in our analysis is the risk of condemning learners to repetition that fails to take understanding forward, and disconnected episodes that rely on processes that are based on memory and/or trial and error.

References

Anghileri, J. 2006. *Teaching number sense* (2nd ed.). London: Continuum.

Askew, M. and Wiliam, D. 1995. *Recent research in mathematics education 5–16*. London: HMSO.

Ball, D.L. and Bass, H. 2009. *With an eye on the mathematical horizon: knowing mathematics for teaching to learners' mathematical futures*. Paper presented at the 43rd Jahrestagung für Didaktik der Mathematik.

Department of Basic Education. 2011a. *Curriculum and Assessment Policy Statement (CAPS): Foundation Phase Mathematics, Grade R–3*.
Department of Basic Education. 2011b. *Curriculum and Assessment Policy Statement (CAPS): Intermediate Phase Mathematics, Grade 4–6*.

Department of Education. 2002. *Revised National Curriculum Statement Grades R-9 (Schools) – Mathematics*.

Department of Education. 2008. *Foundations for Learning Campaign. Government Gazette. Letter to Foundation Phase and Intermediate Phase teachers*.

Ensor, P., Hoadley, U., Jacklin, H., Kuhne, C., Schmitt, E., Lombard, A. *et al*. 2009. Specialising pedagogic text and time in Foundation Phase numeracy classrooms. *Journal of Education,* 47: pp.5–30.

Foucault, M. 1977. *Discipline and punish*. London: Penguin Books.

Gray, E.M. 2008. Compressing the counting process: strength from the flexible interpretation of symbols. In Thompson, I. (Ed.), *Teaching and learning early number*. Second edition. Maidenhead: Open University Press, pp.82–94.

Halliday, M.A.K. and Hasan, R. 1991. *Language, context and text: aspects of language in a social-semiotic perspective*. Oxford: Oxford University Press.

Hasan, R. 2004. The concept of semiotic mediation: perspectives from Bernstein's sociology. In Muller, J., Davies, B. and Morais, A. (Eds), *Reading Bernstein, researching Bernstein*. London: Routledge Falmer, pp.30–43.

Haylock, D. 2006. *Mathematics explained for primary teachers*. London: Sage Publications.

Haylock, D. and Cockburn, A. 2008. *Understanding mathematics for young children: a guide for Foundation Stage and lower primary teachers*. London: Sage Publications.

Hiebert, J. and Lefevre, P. 1986. Conceptual and procedural knowledge in mathematics: an introductory analysis. In Hiebert, J. (Ed.), *Conceptual and procedural knowledge: the case of mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates, pp.1–27.

Hughes, M. 1986. *Children and number: difficulties in learning mathematics.* London: Blackwell Publishing.

Mason, J. and Johnston-Wilder, S. 2004. *Designing and using mathematical tasks*. Milton Keynes: The Open University.

O'Halloran, K. 2000. Classroom discourse in mathematics: a multisemiotic analysis. *Linguistics in Education,* 10(3): pp. 359–388.

Schleppegrell, M.J. 2007. The linguistic challenges of mathematics teaching and learning: a research review. *Reading and Writing Quarterly*. 23: pp.39–159.

Schollar, E. 2008. *Final Report: The primary mathematics research project 2004–2007 – Towards evidence-based educational development in South Africa*. Johannesburg: Eric Schollar and Associates.

Veel, R. 1999. Language, knowledge and authority in school mathematics. In Christie, F. (Ed.), *Pedagogy and the shaping of consciousness: linguistic and social processes*. London: Continuum.

Venkat, H. and Naidoo, D.R. 2012. Analysing coherence for conceptual learning in Grade 2 numeracy lessons. *Education As Change,* 16(1): pp.21–33.

Vygotsky, L.S. 1978. *Mind in society: the development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wertsch, J. V. and Minick, N. 1990. Negotiating sense in the zone of proximal development. In Schwebel, M., Maher. C. and Fagley, N. (Eds), *Promoting cognitive growth over the lifespan*. Hillsdale, NJ: Lawrence Erlbaum, pp.71–88.

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