Teacher learning through teaching and researching: the case of four teacher-researchers in a Masters programme

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Abstract

There is a large body of research and on-going discussions about mathematics teachers' poor content knowledge in South Africa, with many suggesting that teachers need more opportunities to increase their content knowledge. In this article I consider one important opportunity that does not seem to be exploited – that of teachers' learning in the classroom. By considering the learning experiences of four teacher-researchers who were enrolled for a masters degree, I explore how these teachers' mathematics teacher knowledge developed as a result of their research inquiry. The findings indicate that all four teachers have deepened their mathematical knowledge for teaching in the various domains. However, learning in the classroom is enabled by the presence of supportive and knowledgeable colleagues. If authorities want to encourage such forms of learning, then attention needs to be directed to providing intensive classroom support in order to maximise the opportunity for classroom learning.

Introduction

There are many studies that point out the importance of teachers' knowledge in developing students' understanding (Adler, Pournara, Taylor, Thorne and Moletsane, 2009; Ball, Thames and Phelps, 2008; Kriek and Grayson, 2009; Thompson and Thompson, 1994; 1996). In South Africa numerous studies have reported that mathematics teachers struggle with understanding even school level mathematics content. However there seems to be a widespread view that the only way for teachers to improve their knowledge is for them to attend classes or workshops and be taught this knowledge that they need. An important dimension that is sometimes not recognised is that of teachers building up mathematical knowledge by observing and reflecting on the teaching and learning experiences in their classrooms. A constructivist perspective suggests that knowledge is constructed. In the case of mathematical knowledge for teaching, how, where and under what conditions can this construction occur? Recently, Zazkis and Leiken (2010) have focused

on how and what teachers learn through the process of teaching itself.
In this article I look at one particular setting – that of teachers who are engaged in research in their classroom as part of their masters degree studies and I consider how the classroom has acted as a powerful learning site for this group of teachers. In this case, the research process has contributed to the construction of their mathematical knowledge for teaching by providing opportunities for the teachers to engage in detailed observations and critical reflections. In addition, the teachers' engagement with theories and studies related to their own inquiries would have provided additional sources for the reflection, while the supervisors' support and advice would also have facilitated their growth. The purpose of this study is to explore how aspects of their mathematics teacher knowledge developed as an outcome of their research inquiry. This study draws on Ball *et al.*'s (2008) notion of Mathematical Knowledge for Teaching (MKT) which comprises two domains of *Subject Matter for Teaching* and *Pedagogical Content Knowledge*.

Although the participants in this study are a special case, insights gained from this study can help us understand how teacher-learner interactions help develop teachers' knowledge for teaching. Specifically the paper unpacks particular instances of learning that have been prompted by the participants' in-depth observations and reflections around what their learners are saying and doing. Mason (2010) contends that learning through teaching is more likely when teachers observe and reflect on surprising phenomena, and this study provides examples of some of these unexpected phenomena that have prompted the teachers' reflections.

Literature review

One can characterise conceptions of the knowledge required for teaching mathematics as being informed primarily by an internal or external perspective Ernest (1989). An external perspective suggests that mathematics knowledge is acquired from others, while an internal perspective suggests that a person can create their own knowledge. Although it is undisputed that teachers need extensive experience in learning mathematics and learning how to participate in mathematical domains, an internal perspective suggests that valuable teacher learning can occur in the classroom, during teachers' interactions with their learners. Shulman's notion of PCK also makes reference to the fact that much of PCK can originate in the classrooms, when he writes "Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas *others originate in the wisdom of practice*." (Shulman, 1986, p.9, italics author).

The literature on teachers and teaching has identified the need for teachers to be researchers (Roth, 2007). Thompson and Thompson (1996, p.19) recommend that teacher education should focus on building up teachers as researchers, although they concede that "the level of support teachers need to do this far exceeds what most teacher enhancement and teacher education programme provides".

Roth (2007) has no doubt about the fact that teachers who research their own practice become better practitioners. For some teachers "the interest in research may begin with the interest of becoming a better teacher" (p.1). In describing his first research study undertaken as a teacher, he explains that the topic arose from observing the difficulties his students experienced and from his interest in devising strategies that could improve their understanding. Similarly in this article the teacher-researchers' inquiries began with a motivation to learn more about their learners' struggles. In the process these teacher-researchers were able to "learn at quite different levels and about different phenomena" contributing to a strengthening of their mathematics knowledge for teaching (Roth, 2007, p.261). Roth found that as a teacher-researcher, the primary beneficiaries were his school and his learners. Another benefit experienced was that there was no gap between theory and practice – there was no outside researcher telling him what to do or how to change what he was doing.

Thompson and Thompson (1994) in their study of a teacher who struggled to mediate his learners' understanding of the concept of rates found that the teacher had encapsulated the notion of rate within a language of numbers and operations. The teacher's own encapsulation of the concept undermined his effort to help the child develop a conceptual understanding of rate. They suggest that in order for teachers to teach for understanding, they must be sensitive to children's thinking during instruction and to frame their instructional activities accordingly. Thompson and Thompson (1996, p.2), write of the "critical influence of teachers' mathematical understanding on their pedagogical orientations and decisions – on their capacity to pose questions, select tasks, assess students' understanding and make curricular choices".

Based on the abovementioned study Thompson and Thompson (1994; 1996) distinguish between a conceptual and a calculational orientation to teaching mathematics where the second is characterised by "expressing oneself in the language of procedures, numbers and operations" (1996, p.17). They argue further that:

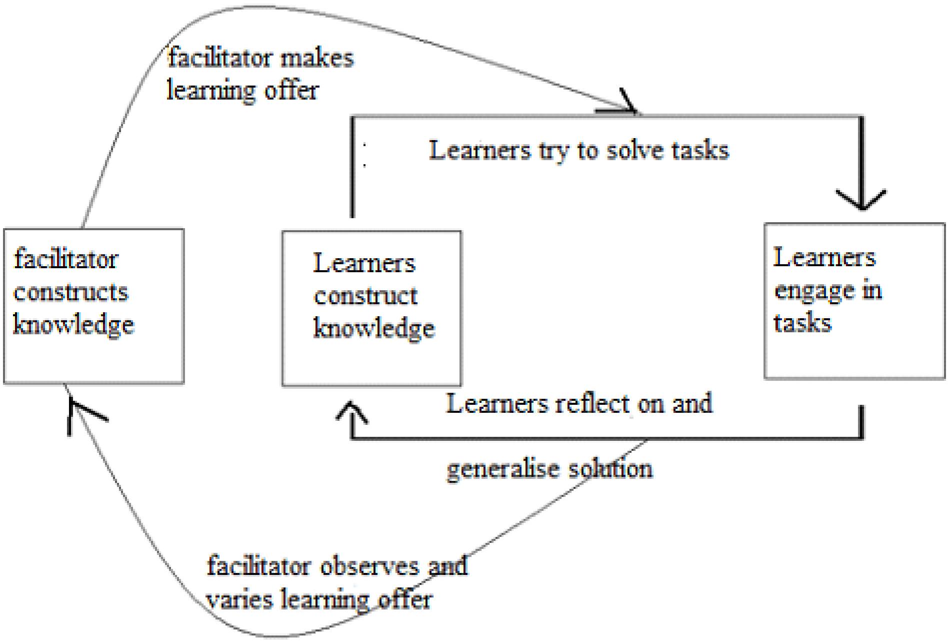
> A teacher with a conceptual orientation is one whose actions are driven by: an image of a *system of ideas* and *ways of thinking* that she intends the students to develop; an image of *how these ideas* and *ways of thinking* can develop; ideas about *features of materials*, *activities*, *expositions and students' engagement with them* that can orient students' attention in productive ways. . . (p.17)

An initial calculational orientation can be shifted and refined into a conceptual orientation by engaging in sustained and reflective work with students and with mathematical ideas (Thompson and Thompson, 1996).

Maoto and Wallace (2006) described how Gerty, a teacher from the Limpopo province was able to move towards teaching for understanding, by learning how to listen. As their study progressed they found that Gerty spent more time trying to make sense of her learners learning. As Gerty tried to examine the ideas underlying her learners' confusion, she strengthened her own understanding of the mathematics she was teaching. By trying to design or adapt activities that met her learners' needs she further developed this understanding. In trying to identify the basis for some of her learners' misconceptions she had to grapple with many of the underlying mathematical ideas. In this way, her observations of, and reflections on her learners' struggles helped her improve her own understanding of what it meant to teach for her learners' understanding. It is important to note that Gerty's understanding was enhanced because of the presence of a supportive colleague with whom she could discuss her concerns, doubts and her growing awareness of teaching for learners' understanding. Similarly Davis (1997) found that the key site for the teachers' learning was the classroom itself, and the learning was facilitated by the presence of a supportive colleague.

Thompson and Thompson (1994, p.280) argue that the relationship between a teachers' and a student's ways of knowing is a reflexive one. This suggests that the teachers' way of knowing contributes to the students' development of understanding. In response, as students develop their understanding, their interactions with the teacher influences the way in which the teacher understands a concept. Somewhat related to this reflexive process is the model of Steinbring (1998, p.159) which describes how learning by both pupils and teachers can be enhanced during a mathematics lesson. The learning of pupils and teachers, although autonomous, are linked to each other and build on each other. Steinbring emphasises the role of reflection in the learning of both pupil and teacher. Zaslavsky (2009) offers a modification of Steinbring's model which is reproduced here.

Figure 1: Steinbring's model of Teaching and Learning as modified by Zaslavsky (2009, p.107)



This figure illustrates that the teacher's (facilitator's) learning is "an outcome of their observations of learners' engagements in tasks" and their reflections on learners' work (Zaslavsky, 2009, p.107). Learners learn by engaging in a task, and also by trying to reflect on, and generalise the solutions. The teacher in turn, observes the process learners are engaged in, tries to vary the learning offers, and also reflects, which leads to his/her own learning. There are two loops of learning that are represented in this adapted model, one showing the learning by reflection of the learners and a second loop showing the learning of the teacher by reflection and observation of the process encountered by the learners. Steinbring (1998) notes:

> Students' mathematical knowledge is more personal, is bound to special exemplary contexts, and is in the process of an open development ... The mathematics teacher has to become aware of the specific epistemological status of the students' mathematical knowledge. The teacher has to be able to diagnose and analyse students' construction of mathematical knowledge . . . to vary the learning offers accordingly (p.159).

Central to this construction of knowledge by the teacher, is the process of reflection which facilitates the process. Thompson and Thompson (1996) suggest that teachers can come to understand a mathematical idea, in a way that enables them to teach it conceptually

> through sustained and reflective work with students and with mathematical ideas – comparing their attempts to influence students' thinking with disinterested analyses of what those students actually learn – and reflecting on both what they intended and what (they understand) they achieved (p.19).

Kraft (2002) asserts that good teacher research must centrally involve self-critical examination of those belief systems that inform and guide practice in the first place. Rosenberg (2008) has usefully summarised Hatton and Smith's (1995) description of four levels of reflective writing as a means of thinking about teachers reflections:

1. *Descriptive writing* (writer reports on events with no reflection on them at all);

2. *Descriptive reflection* (writer provides reasons for pedagogical decisions based on personal judgement);

3. *Dialogic reflection* (writer explores the reasons for one's pedagogical choices in the light of educational theory); and

4. *Critical reflection* (involving reasons given for decisions or events that take into account broader historical, social and political contexts).

For Brookfield (1995), critical reflection involves 'hunting' the assumptions that underpin our teaching practices. This process involves questioning what we take for granted and what we do in order to make our teaching lives easier, but which may not be working in the way that we assume. Paradigmatic assumptions are the hardest of all assumptions to uncover, because, as Brookfield explains,

> [t]hey are the basic structuring axioms we use to order the world into fundamental categories' and we seldom recognise them as assumptions, even after they have been pointed out to us. Instead, 'we insist that they're objectively valid renderings of reality, the facts as we know them to be true' (1995, p.2).

Having looked at some research about teachers' learning in their classrooms, I will now briefly discuss the framework of teacher knowledge that underpins this study.

Framework

The field of teacher knowledge is a vast one with many researchers generating descriptions and definitions which try to capture exactly the kind of knowledge that is needed to mediate learning in the classroom. Ball *et al*., (2008, p.395) use the term "Mathematical Knowledge for Teaching" to refer to "the mathematical knowledge needed to carry out the work of teaching mathematics". Their perspective is that Mathematical Knowledge for Teaching (MKT) comprises two domains which are *Subject Matter for Teaching* and *Pedagogical Content Knowledge*. Subject matter for teaching has been further divided into two subdomains of *common content knowledge* which is "mathematical knowledge and skill used in settings other than teaching" and *specialised content knowledge* ie., "mathematics knowledge and skill unique to teaching" (Ball *et al*., pp. 399–400). They provide examples of common content knowledge as when teachers know the work that they assign to their learners, or using terms and notation correctly, being able to recognise learners' wrong answers. Specialised content knowledge is beyond the knowledge taught to learners and includes "understanding different interpretations of the operations in ways which students need not explicitly distinguish" (p.400). Ball *et al.* also postulate a provisional third subdomain called *horizon knowledge* which "is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p.403).

In Ball *et al.*'s model pedagogical content knowledge is divided into two further subdomains. The first is *knowledge of content and students* ie., knowledge that permits an "interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (p.401). The second subdomain is *knowledge of content and teaching* i.e. knowledge that allows "an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (p.401). The authors have provisionally identified a third subdomain *knowledge of content and curriculum* which roughly coincides with Shulman's (1986, p.10) category of curricular knowledge which

> is represented by the full range of programs designed for the teaching of particular subjects and topics at, a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances.

Now that the framework used to illustrate the concept of mathematical knowledge for teaching has been elucidated, the details of the methodology of the study will be discussed.

Methodology

This is a qualitative study, utilising a case study approach in order to capture part of the storied lives of the four participant teachers, who were researching a problem they identified as part of their Master's degree studies. The participants will be referred to as teacher-researchers or as teachers, depending on which of these roles is under discussion. Data was generated from the thesis reports of the teacher-researchers, and my own field notes (as their research supervisor) drawn up during the approximately 150 hours that I spent with each of the participants during their research. The process followed in constructing the vignettes is what Polkinghorne (1995) described as narrative analysis. During the process of analysis, outcomes were roughly identified, from the data. Thereafter I went back and forth from the data to the emerging descriptions building up and identifying "thematic threads" relating to those outcomes (Polkinghorne, 1995, p.12). The happenings were then configured or 'emplotted' to take on a narrative meaning – they are understood from the perspective of their contribution and influence on the teacher-researchers' construction of mathematics knowledge for teaching. That is, each vignette is a reconstruction of events and actions that produced the particular outcomes. The second stage was to then use an "analysis of narratives" (Polkinghorne, 1995, p.12) technique to find best fit categories that were consistent with Ball *et al.*'s (2008) mathematics knowledge for teaching framework. The purpose of the analysis was to provide answers to the central research question: How does the research inquiry process contribute to the construction of mathematics teacher knowledge by the teacher- researcher?

Results

In this section I present vignettes of each of the four teacher- researchers drawn from the data.

Learning about learners' assessment feedback expectations

Naidoo, a Grade 9 Mathematics teacher, was concerned by the poor performance and the high failure rates of learners in mathematics (Naidoo, 2007) and raised questions about the effectiveness of her feedback in her mathematics classroom. She was concerned about whether feedback was communicated in a way that was useful to learners. She also wanted to find out whether certain forms of feedback information were more useful than others. Naidoo's concern about the effectiveness of feedback arose from her observations of learners who often repeated the mistakes even after receiving feedback on how their mistakes could be corrected (Naidoo, 2007).
Naidoo used journals and interviews to probe the learners in her class about their opinions of assessment feedback.

She found that some answers she received were sometimes supported and at other times contradicted by findings reported in the literature. Naidoo was surprised by the depth of the learners' understanding of assessment. She found that her learners' understanding of teacher feedback "provided insights into the value and purposes of feedback" as identified by researchers and educationists (Naidoo, 2007, p.87). She found in the interviews that learners wanted guidance on how they could improve, and comments such as 'good' or 'excellent', were not seen as particularly helpful to them. The learners also spoke about the negative effect that some comments had on them. This view is supported by Young (2000, p.4) who wrote that verbal comments that are derogatory are viewed as "absolutely annihilating" (Young, 2000, p.414) for the learner in the learning experience. Other findings about the conceptions held by learners about the role of feedback in providing directions on how to proceed with a calculation, improving understanding of a mathematics concept and filling in gaps in understanding were supported by research findings from other countries.

A finding not reported in literature, was the value of written feedback because they could "read and make sense of what is written at a later stage", thereby constituting a resource for further learning (Naidoo, 2007, p.94). There were other findings that were different from what was expected. The study identified a learner who did not welcome her teachers' unsolicited feedback, especially if she was busy concentrating on her work. She wrote that she was irritated if the teacher disturbed her concentration just to mark her work. The answers that Naidoo received were not always expected but this enabled her

to develop a personal knowledge of her learners' expectations of the teacher's role in helping them develop their mathematical understanding.

Unpacking learners' calculations

Khan, a Grade 9 mathematics teacher was concerned that her learners often performed badly in the Common Tasks for Assessment (CTA), yet she noticed that their performance in the classroom based Continuous Assessment (CA) component was reasonable. The means of the class for the average of the CA and the CTA were 58 and 34 respectively, a 24 percentage point difference, showing that on average learners only achieved 58% of their CA marks in the CTA.

Her concern over the discrepancies in performance between the two assessments led her to investigate why her learners performed more poorly in the CTA than in the CA (Khan, 2009). She found that the task design of setting each sub-task within one extended context, was problematic for the learners. The learners struggled with resultant language load used to portray the context. The learners struggled to identify the crucial information from all the extraneous details associated with the contextualisation. The readability levels of certain instructions were beyond the average Grade 9 level (when measured using an easily available readability index).

Khan also did analyses of errors similar to that described by Ball *et al*., (2008, pp. 397, 400) as being characteristic of the distinctive work done by mathematic teachers and which forms part of what they term specialised content knowledge. The figure below consists of a task followed by a learner's (Cleo) written response.

Figure 2: A learners' response to a contextualised task (DoE, 2005, p.6)

Activity 1.4 | **Individual**

Recommended time: 30 min | **Marks: 17**

1.4.1 (a) Use the map on page 4 to estimate the distance from your school to the marked entrance to the Kruger National Park. (5)

(b) Why can the answer in (a) only be an estimation? Give two reasons. (2)

1.4.2 When the CTA was copied at a certain school the map on page 4 was reduced to 80%. What influence can this have on your answer in 1.4.1? Explain your answer. (3)

1.4.3 Up to 1994 the KNP stretched 350 km along the Mozambican border and was on average 60 km wide. Use this information to determine the approximate area of the Park before 1994, in km^2. (2)

1.4.4 According to one source, the actual area of the Kruger National Park before 1994 was 2 149 700 hectares. Convert your answer from 1.4.3 to hectares and explain why your answer differs from the actual area. (2)

1.4.5 According to an agreement with the governments of Mozambique and Zimbabwe, the Kruger National Park will become part of the Greater Limpopo Transfrontier Park. The eventual size of this Park will be 100 000 km^2. Calculate the percentage increase of this Park compared to the size of the Kruger National Park before 1994 (2 149 700 ha). (3)

Activity 1.4

1.4.1. a) 40 x 16 2
= 640 – The distance from my school.

b) 1) It because I draw a straight line
2) The map is also showing

1.4.2 The answer Can Change because the distance Can also change.

1.4.3 ~~/ 410 is the final amount in 1994 distance)~~ 350 x 60 = 21,000

1.4.4 168100 the actual answer was 410 and I Convert it.

1.4.5. 21497 / 462,12 1009 the is the increasing mark. 3

In trying to understand the learners' response to Question 1.4.4, Khan inferred:

> For Question 1.4.4, Cleo extracted the numbers 350 and 60 (representing the length and width of the Park) from Question1.4.3. She proceeded to add the length and width obtaining the number 410, which she then squared to obtain a value of 168100. It is evident that Cleo was struggling to make sense of question 1.4.4. The instruction "*Convert your answer from 1.4.3*" seems to have cued her to extract the numbers 350 and 60 from 1.4.3. Furthermore her comment "*the actual answer was 410 and I converted it*" reveals that she interpreted the instruction "Convert your answer" to mean, "*square the result*".

For Question 1.4.5. Cleo responded by writing 21497/462,121009. It is a fascinating exercise to trace how she arrived at these figures. Firstly, Cleo extracted the 2149700 (representing the original area in hectares) and divided it by 100 to arrive at the 21497 in the numerator. To get the number in the denominator, she extracted the only two figures of 2149700 and 10 000 (representing the original area in ha. and new area in km^2), which appeared within the maze of instructions. She then divided the first number by the second and squared the result, to obtain the number 462,121009 in the denominator. This provides an indication to us, of Cleo's struggles to provide answers to questions that she did not understand (Khan and Bansilal, 2010, p.290).

Khan's error analysis was done to take her beyond seeing "answers as simply wrong" but it is an attempt to gain a "detailed mathematical understanding required for a skilful treatment of the problems students face" (Ball *et al.*, 2008, p.397). In this case the problem was the misinterpretation of the instructions and therefore the role of the language in hampering the learner's attempt at the problem. Thus Khan's (2009) study challenged her views about the use of contextualised tasks in controlled settings. When she began the study, she did not question that a national assessment protocol could be unreliable. Her experiences led to a broadening of her understanding of the need for validity, consistency and reliability of assessments, especially in a diverse context as South Africa. Her findings led her to take a critical stance, and she concluded that the CTA programme of assessment was not a fair means of assessment. She further recommended that the design should be urgently revised.

Reasoning about the rules of differentiation

A third case is that of Pillay, a Grade 12 mathematics teacher, who was concerned over the years about her learners' lack of conceptual understanding. This concern led her to design a study to explore her Grade 12 learners' understanding of the concept of the derivative. She analysed the written responses of her class to two assessments, and interviewed four learners in a bid to understand the ways in which they responded to the tasks. By turning her attention to the notation errors made by her learners, she was able to deepen her own understanding of many of the conventions used to denote derivatives and functions. In the process of studying her learners' misuse of rules, she deepened her grasp of the relationship between the various rules of differentiation. She also observed that the presentation of a question could cue learners to produce certain responses. The learners' tendency to draw on formulae from quadratic theory, compelled her to investigate links between and within the cubic and quadratic functions. These reflections shifted and transformed her own 'big ideas' of the purpose and value of certain algorithms.

Pillay's understanding of learning theories such as mathematical proficiency (Kilpatrick, Swafford and Findell, 2001) deepened. She was able to find evidence, for example, of students' lack of procedural fluency when carrying out certain rules, while also identifying instances of learners who demonstrated conceptual understanding. She found evidence that many learners knew how to perform a procedure but did not know when to use it, such as when they used the turning point formula for finding the turning point of a quadratic function, to find the turning point of a cubic function. Many learners, when asked to find the value of the derivative of function at a certain point, found the value of the function at that point.

Her study led to a deeper understanding of links between concepts such as gradient and derivative. Her understanding of the rationale behind the sequencing and approach to certain topics was also strengthened. She was able to see the reasoning behind the introduction of calculations of the average gradient between two points on a graph a year before the introduction of the concept of the derivative.

> One of the assessment standards at grade eleven requires learners to investigate numerically the average gradient between two points and thereby develop an intuitive understanding of the concept of the gradient of a curve at a point. In the grade twelve year learners are required to investigate and use instantaneous rate of change. They are required to

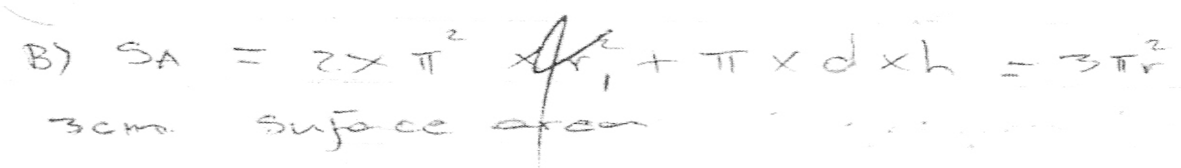
> accomplish this by first developing an intuitive understanding of the limit concept in the context of approximating the gradient of a function. This sets the stage for establishing the derivatives of various functions from first principles. With the emphasis now being on the concept of average gradient, leading to the concept of the derivative it is hoped that learners will develop a better understanding of the concept of the derivative (Pillay, 2009, p.108).

The findings of her study led her to recognise that curriculum change alone may not produce the desired conceptual understanding. Her recommendation was that curriculum change should be accompanied with a change in the type of assessment tasks that are given to learners, in order to foster conceptual understanding of the derivative. She also highlighted the role of sequencing in influencing learners' understanding. The phrases 'met-before' and 'met-after' were coined by Tall, cited in De Lima and Tall (2008), in order to describe the effect that prior learning can have on newly introduced concepts. Pillay's study revealed to her that the algorithms linked to the quadratic functions (such as calculations of the roots of a quadratic equation, turning point of a quadratic function) sometimes impacted negatively as a met-before for the algorithms linked to cubic functions. This recognition of the links between the two topics, led her to extend and revise her own big ideas of graphs of quadratic and cubic functions.

Re-thinking the purpose and form of assessments in Mathematical Literacy

A fourth case is that of Debba (2012) whose concern about his learners' disengagement with contexts used in Mathematical Literacy (ML) assessment tasks, led him to try to identify reasons behind the disengagement. Some of his findings were unexpected and contradicted previous research showing that if students understood certain contexts they were more likely to perform better. In certain cases, learners' previous experiences of the context influenced their responses differently from what was expected from them. He also found that many of his learners were not interested in making sense of the contexts but were more interested in picking out formulae to generate answers but did not necessarily use the formula productively. For example, the excerpt below shows a learner's response to a question which asked for the circumference of a tool whose dimensions were represented in a diagram.

His response when scrutinised shows that the learner did not substitute any values into the formula that was provided, but simplified the expression by ignoring the variables. He responded similarly to a second question which asked for the surface area of a figure:



Here too, it is clear that the learner did not substitute any value but just wrote an answer, showing again that he did not know what to do with the variables. This tendency to ignore variables in simplification of algebraic expressions is a common misconception which often manifests itself in early algebraic experiences.

Debba progressed from believing that it was unfair on learners to use contexts which were unfamiliar to them in the ML classroom, towards distinguishing between the purposes of the different contexts. His observations and reflections allowed him to distinguish between the use of contexts for developing understanding in mathematics as opposed to preparing learners to make informed decisions. His study led him to see the importance of using authentic life related contexts with learners in order to fulfil the mandate of ML, irrespective of whether they were familiar or not. This progression in the understanding of the important role played by authentic contexts changed Debba's views of ML assessments, convincing him that examinations actually detract from the fulfilment of ML aims. He argued that the learners need opportunities to engage in authentic contexts so that when they encounter these, they can make informed decisions as is the mandate of ML. However setting examination tasks around contexts which they have not encountered before, disadvantages many learners if they miss crucial information or are unable to understand certain conventions about the context. He wrote:

> Hence if we want learners to engage with life related issues, we need to find alternate means of assessment rather than using examinations only. The examination setting creates pressure on learners to pass, and not to bother about making sense of what is being asked. The examination setting also restricts the kinds of authentic engagement that could help learners get to grips with real life issues (Debba, 2012, p.117).

Thus his 'big ideas' about the role of contexts in an ML classroom shifted in conjunction with his shift in understanding of the purpose and rationale behind ML.

Discussion and concluding remarks

In this article I have presented a case of four teachers who by doing a systematic inquiry into a learning issue that they were concerned with, strengthened their own mathematics knowledge for teaching. We found that the teachers' knowledge of their learners, knowledge of the effect of sequencing, knowledge of the content, and knowledge of the curriculum were deepened.

In the case of Naidoo, by finding about her learners' understanding and preference for certain types of assessment feedback in her mathematics class (which was sometimes different from what she expected), she increased her knowledge about her learners. Her insights into the value of feedback has influenced her understanding of assessment which has allowed growth in pedagogical knowledge, which is however not a focus of the framework of Ball *et al.* (2008) but is an important knowledge area for a teacher. A significant point is that the domain of her personal knowledge of teaching has also been enlarged, by the insights she has received. Her future offerings of feedback to her learners will never be the same as it was before the study. A further point from the case of Naidoo, is that her findings were sometimes different from that reported in the literature, and her knowledge of learners originated in 'the wisdom of practice' (Shulman, 1986), and was further enhanced by the activities she engaged in as part of her research.

In the case of Khan, by engaging in the error analyses of her learners, she has enlarged her domain of *specialised content knowledge*. Khan by her in-depth analysis of learners' responses had also increased her *knowledge of content and students* by hearing and interpreting her learners' emerging thinking and their opinions and frustrations of the language overload in the tasks. Furthermore she was able to engage in critical reflections by *hunting her paradigmatic assumptions* (Brookfield, 1995) that national assessments are valid and reliable instruments that can indicate whether learners know and understand their work. She has learnt that the use of the extended context in the CTA programme, actually led to the instruments being unreliable and unfair. She has been able to take a critical stance about validity issues in assessment.

In the case of Pillay, by looking at the learner's misconceptions in using inappropriate algorithms in calculating turning points she has been able to develop her *common content knowledge* in graphs of quadratic and cubic functions by looking at similarities and differences between the two. She has also articulated her understandings in the domain of *horizon content knowledge* by engaging in discussion of how the rules of differentiation are related to other techniques of differentiation and by seeing how the approach to the teaching of calculus has been developed in earlier years. This understanding of the linkages and sequencing of concepts in the curriculum is also part of *knowledge of content and curriculum* and can be described as vertical curriculum knowledge (Shulman, 1986, p.10).

Debba's study helped him develop an understanding of the curriculum of ML, that is he has deepened his understanding of *knowledge of content and curriculum* by re-examining his big ideas about the purpose of ML. He too, has improved his *knowledge of content and student,* because he has learnt about his learners' misconceptions as well as their approach to providing answers to the assessment tasks. Debba has also hunted his *paradigmatic assumptions* that familiarity of contexts is necessary for learners in order to help them develop ML skills, and has instead realised that teaching in the subject ML should provide opportunities for learners to engage in the unfamiliar contexts. This critical reflection has enabled him to take a critical stance on the issue of the form of assessments used in ML, and he recommends that assessment should not be done by examinations because the limited time frames mean that learners do not engage with the context, thereby actually constraining the vision of ML.

Thus in all four cases, we have seen that these teacher-researchers have deepened their mathematical knowledge for teaching in different domains, by engaging in inquiry in their own classrooms. In all these cases, a precondition for this learning was the critical reflection that they engaged in, which was facilitated by their research inquiry process. However, these teachers did not simply start to reflect on their teaching, but did so in a formal systematic way while being guided by their supervisor. Although the teachers' learning was an outcome of the research process, their detailed observations and willingness to hunt their framing assumptions enabled their learning. The teachers, as part of the research process also engaged with theory and critically assessed other research studies related to their own inquiries, and these activities would also have enhanced their learning.

The question then arises whether teachers can only learn if they are enrolled for postgraduate studies, as in the case of this sample. Are there other teachers who find that the classroom can be a powerful site for their own mathematical knowledge for teaching? Can learning occur outside of a research setting? Clearly the answer to this question is yes — like in the case of Gerty (Maoto and Wallace, 2006) where learning occurred in a research setting although she was not the researcher. A further enabling factor seems to be the presence of a concerned 'other' who is able to listen to the teachers articulate their learning while providing a non-judgemental ear. In the four cases presented in this article, the research supervisor would have played that role. In the case of Gerty it was the researcher, while in the case of Thompson and Thompson (1994; 1996), the researcher also supported the teacher in his struggles to cross the gap between his own understanding and that of his learner. A study by Peressini and Knuth (1998) described how a teacher struggled to understand a different solution method offered by a group of students. It took the intervention of a pre-service teacher to convince the teacher of the equivalence between the solutions. Thus in this case, the support to the experienced teacher was provided by a novice teacher.

In South Africa, little attention has been paid to implementing classroom teacher support. Instead there has been a focus of external interventions, which aim to 'increase' teachers' knowledge. Current discourses in South Africa seem to ignore the classroom dimension of teacher learning. Ministers, journalists, as well as Non-Governmental Organisations often speak about teachers' poor content knowledge but ask for interventions in the form of formal courses to improve the knowledge. Teachers themselves ask for more knowledge interventions, showing that they believe that they can only learn when others teach. However this article has demonstrated that the classroom can be a powerful site for teachers to improve their own knowledge. It has shown that for teachers who set out on a disciplined inquiry centred around teaching and learning issues in their classroom, one of the benefits is a deepening and shifting of their own mathematics knowledge for teaching.

Perhaps teachers themselves need to be convinced about this. There is a whole generation of teachers who have lost faith in their ability to learn by themselves and will need sustained support in order to start taking steps in their own professional growth. Bansilal and Rosenberg (2011) in their study of 41 teachers' reflective reports found little evidence of teachers reflecting on learners' learning. Many of the teachers engaged in descriptive writing and not critical reflections, suggesting that teachers will need much support in order to encourage them to take on their own learning by observing, reflecting, sharing and learning. Research (Davis, 1997; Clark and Linder, 2006; Maoto and Wallace, 2006) emphasises that a collegial and supportive environment is crucial to teachers' growth, comfort and well-being. Thompson and Thompson (1996, p.19) have also cautioned that the level of support teachers need in order to develop such a conceptual orientation to teacher knowledge "far exceeds what most teacher enhancement and teacher education programme provides". However such a model that helps teachers

take responsibility for their own learning should be prioritised by education authorities because of the immense benefits it offers.

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