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# Editorial

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## Carol Bertram and Iben Maj Christiansen

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### Abstract

There has been a huge growth in research on how to describe the kind of knowledge that teachers need since the mid-1980s when Shulman (Shulman, 1986, 1987) published his first research in this field. There is a range of ways in which to describe the field of teacher knowledge. In this editorial we aim to describe one way of mapping the field, and then place the papers in this Special Issue as well as other recent work onto this map. We do so by proposing that the propositional, the practical and the personal are different aspects of teacher learning, and that the relations between these demand more attention. We then discuss how the common categorisation of teacher knowledge can be viewed within each of these aspects, and how the key may lie in the relations as well as in the categories themselves. Finally, we reflect on the ‘field’ of teacher education research on the basis of the range of submissions we received. An outstanding feature of the field is its weak grammaticality, meaning that there are a range of concepts in use which do not seem to have very precise empirical descriptions (Bernstein, 1999), but this appears to vary from discipline to discipline suggesting directions for further work.

### Three aspects of teacher learning: knowing, doing, being

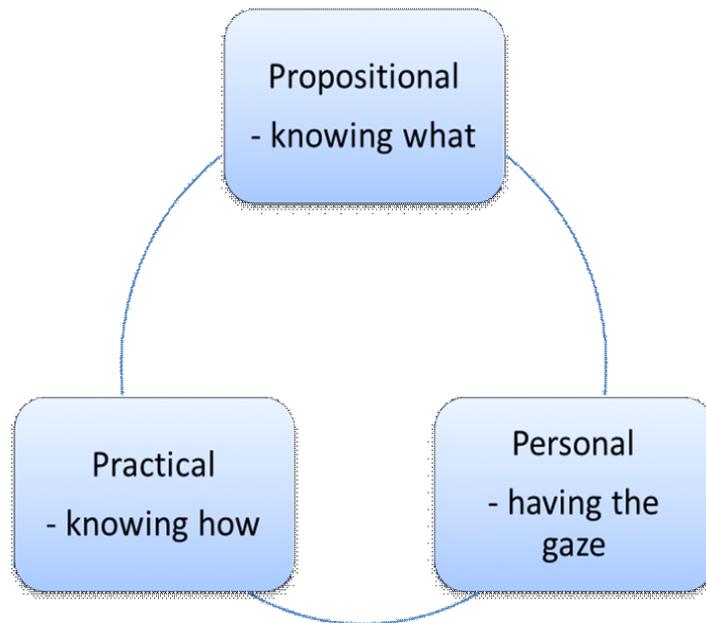
In their attempt to map the field, Cochran-Smith and Lytle (1999) argue that there are three broad approaches to teacher learning, which they then describe (somewhat confusingly) as ‘knowledges’. They refer to the first conception of teacher learning as *knowledge-for-practice* where the assumption is that researchers generate a formal body of knowledge which teachers can learn and use to improve their practice. They describe a second conception of teacher learning as *knowledge-in-practice* which is essentially practical knowledge generated by expert teachers through their own experience and practice. The third conception of teacher learning involves *knowledge-of-practice* which is the knowledge produced when teachers treat their classrooms and schools as sites of intentional investigation. It implies a critical stance to one’s own practice, and thus more strongly than the other conceptions relate to subjective theories of teaching and learning, as well as to beliefs about oneself, purposes

of teaching, etc. The two latter orientations honour the idea that teachers often learn in and from practice, and not exclusively from a formally generated, codified body of knowledge.

Another way of describing the kinds of knowledge that are foregrounded in these different conceptions of teacher learning is to say that knowledge-for-practice privileges propositional knowledge, codified knowledge, or knowing 'that', whereas knowledge-in-practice privileges practical knowledge or knowing 'how' (cf. Tamir, 1988). The 'knowledge-of-practice' conception seems to foreground the inquiry stance that a teacher takes, and is interested in how teacher inquiry generates knowledge, how inquiry relates to practice, and what teachers learn through inquiry. Using Maton, we could say that this orientation foregrounds a knower code or the social relation, but as Maton always says, this does not mean that the knowledge code or the epistemic relation is non-existent, since for every knowledge structure, there is also a knower structure (Maton, 2007). In line with this, we will not include perspectives on teacher learning which claims that the epistemic relation is of no relevance.

We believe that the three orientations should be seen not as mutually exclusive but rather as foregrounding different aspects of teacher knowing: the propositional, the practical and the personal. All three of these aspects are necessary in professional practice. Fischer (2011) has a different third aspect, namely conditional knowledge or 'knowing why', engaging in the project of interrogating instructional quality. We see this contained more in the concept of inference and professional judgement, which we will return to later.

**Figure 1: The three aspects of teacher knowledge**



A key area of engagement is the question of how the aspects relate to one another. In South Africa, the post-1994 curriculum reform and qualification frameworks foregrounded practice and what people could do (their competence) while knowledge was backgrounded. Although the Norms and Standards for Educators described foundational competence as demonstrating an understanding of the knowledge and thinking that underpins a particular action, this was often reduced to simply a ‘skill’. Unfortunately, an unintended consequence of the implementation of the policy was that many teacher educators fixated on the ‘seven roles of educators’, and designed their teacher education curricula around the seven roles, and not around the competences which incorporate both principled knowledge and thoughtful practical knowledge (Department of Higher Education and Training, 2011).

However we know that action is underpinned by thought, and that some action is more knowledgeable and thoughtful than other action (Muller, 2012). Since teaching is a professional practice, it is informed by both ‘knowing what’ and ‘knowing how’, but also by the motives, beliefs, disciplinary philosophies of the teacher (Langford and Huntley, 1999; Lloyd, 1999; Thompson, 1984). It is through engaging with the ‘knowing what’ and ‘knowing how’ of the disciplines that beliefs and disciplinary philosophies may become more nuanced, consistent and informed – or at least conscious.

They relate, and it is in this relation that we see teacher knowledge positioned. The paper in this issue by Venkat and Askew discuss the case of teachers who

were given new mathematics resources (such as an abacus) to use, but who failed to do so in a way which foregrounded key mathematical notions (in this case, the importance of focusing on the grouping into tens reflected in our number system). We could view this as the teachers lacking either the propositional mathematical or pedagogical content knowledge which could have guided their professional judgement. That is certainly a possibility, though one that Venkat and Askew reject. They propose instead that it is a consequence of absence in the sociocultural setting – the teachers have not previously encountered such uses of these resources. This illustrates how, in order to use the abacus to teach the key mathematical notions, the teachers have to have the propositional mathematical and pedagogical content knowledge, the practical knowledge of how to use resources constructively, the ability to infer from the one to the other to be able to recognise and realise the use of the new resources, and an orientation to teaching which directs them to bring this into play.

In South Africa we have perhaps – particularly in recent times – shunned the ‘knowing what’ because we have tended to equate it with lists of unconnected information, and have not focused sufficiently on the conceptual relations between all these facts. So we teach ‘lists’ rather than principles, but lists do not support the development of professional judgement. And as Ensor’s work on student teachers’ recontextualisation from teacher education to practice in school indicates, neither does propositional knowledge which is not linked to practice (Ensor, 2001). Thus we propose to focus on this gap that opens between the ‘knowing what’ and the ‘knowing how’ both in teacher education and as the space where research work needs to happen.

This link between the propositional and the practical, however, also draws on the particular gaze that the teacher has developed in creating an identity as a teacher, as developed in a sociocultural setting. Similarly, the link between the propositional and the practical is what helps to construct the particular trained teacher gaze that allows for recognition in practice of instances from theory, and informs professional judgement. It is this gaze which allows a teacher to recognise what counts as valid inferences in practice, drawing on knowledge for and about practice (cf. Muller, 2012). And it is this gaze which informs the teacher educator’s choice of the propositional and the practical to include in teacher education.

## Five domains of teacher knowledge

We move one level down from these broad perspectives to engage the five domains of teacher knowledge widely used: content knowledge, general pedagogic knowledge, pedagogical content knowledge, curriculum knowledge and knowledge of context (3 C's, GPK, and PCK!). The assumption that these are separate knowledge domains has been challenged, most recently in an Australian study of mathematics teachers' knowledge and beliefs (Beswick, Callingham and Watson, 2011), which claims that their results indicate one underlying knowledge dimension only. Nonetheless, we find the distinctions analytically useful.

Shulman has been critiqued that his categories are propositional and do not account for practical knowledge (Jones and Straker, 2006). However, in our view, all of these domains of knowledge have both propositional and procedural, as well as personal, elements.

Content knowledge embraces both the propositional knowledge/'knowing what' and the procedural knowledge/'knowing how' of a discipline. Teachers need to know more than just the 'facts' of their discipline, they need to know the deep underlying principles and structure of the discipline, and they need to know what procedures are used to generate knowledge in the field. Muller (2012) argues that 'knowing what' comes down to knowing why something is accepted as knowledge in the relevant field, but this implies knowing how to substantiate the knowledge, knowing how to make such arguments, so that all 'knowing what' also comes down to the particular 'knowing-how' of drawing inferences within the field. Coming together in a learned gaze. . . The historian does not separate looking at a particular event with an awareness of the time in which it happens and how it relates to what went before, from critically looking at the sources which provide information about the event. A mathematician does not separate knowing what the boundaries of a concept are from knowing how to use this in constructing a proof.

In the German COACTIV-study (Baumert, Blum and Neubrand, 2004; Krauss and Blum, this issue), PCK in mathematics is divided into two facets: declarative and procedural PCK. The declarative encompasses knowledge of learners' common misconceptions, knowledge of the curriculum, and predicted difficulties. The procedural includes selection of tasks, reacting to students, and assignment of homework (Olszewski, Neumann and Fischer, 2010). In some studies, the procedural PCK is explored through 'situational

judgement tests' or 'teaching vignettes' (cf. Riese and Reinhold, 2010 for physics education) while others consider using it in video analysis (Olszewski, Neumann and Fischer, 2010 for physics education; Ramdhany, 2010 for mathematics education). Fischer (2011) found a correlation between declarative and procedural pedagogical knowledge of 0.20 ( $p < 0.01$ ) and thus see them as independent constructs, while Riese and Reinhold (2010) found correlations between 0.64 and 0.84 ( $p < 0.001$ ), stronger than the correlations they found between PCK and content knowledge, PCK and pedagogical knowledge, and between content knowledge and pedagogical knowledge. All these studies confirmed that content knowledge is however a prerequisite for PCK, as also suggested by the study by Beswick, Callingham and Watson (2011).

An outstanding feature of the field of teacher education is its weak grammaticality, meaning that there are a range of concepts in use which do not seem to have very precise empirical descriptions (Bernstein, 1999). As a clear example of this, every study on PCK has to operationalise the concept, and often does so in slightly different ways. This issue is critically engaged in Adler and Patahuddin's paper in this issue. Working with the notion of 'Mathematics for Teaching,' which positions itself in relation to PCK and content knowledge, they explore how carefully designed test items can facilitate teachers' talk and mathematical reasoning, leading to an exploration of knowledge connected around the teaching of specific content. Thus, their work engages both relations between knowledge domains, connections of aspects, and issues of how to address weak grammaticality in the field.

This distinction between declarative and procedural PCK becomes highly relevant when wanting to see if teachers who are able to respond well to PCK questions (which often assess what in the above distinction would be propositional PCK such as identifying learner misconceptions or levels of learner thinking) are also able to apply this knowledge in the classroom in ways which enhance learning. The paper by Krauss and Blum in this issue summarise their findings from the COACTIV project. Utilising new instruments which are open-ended rather than the oft used multiple choice approach, they explored teachers' content knowledge and PCK with practice-related vignettes, and the impact of teachers' knowledge on learners' achievements. Their findings show that content knowledge and PCK are linked to beliefs as well as practice, for us highlighting the relation between the three knowledge aspects. Their study found PCK but not content knowledge linked to learner achievement. This does not mean that content

knowledge does not matter – but it is possibly doing so only indirectly, as content knowledge is correlated with PCK (Riese and Reinhold, 2010), and appears to be a prerequisite for PCK but not implying it (Riese and Reinhold, 2010). The latter also appears to hold in South Africa, though no clear link between PCK and learner achievement was found here (Ramdhany, 2010). Finally, they found no link between number of years of experience and demonstrated PCK, but this must be seen in light of the fact that there were no novice teachers in their study.

There is, as in all professions, a very real discursive gap between the theoretical and the practical (Muller, 2012), between knowing what errors learners often make and being able to use that knowledge to make professional judgements in the classroom, between having the academic knowledge and having what Shalem (forthcoming) using Abbott's work refers to as the diagnostic knowledge (cf. Ashlock, 2002; Cooper, 2009)

Two processes are involved in diagnostic knowledge. First, the practitioner collects information about a particular case . . . and assembles it into a complex picture, according to certain epistemic rules and criteria specific to the subject matter. Second, the practitioner takes the complex picture and refers it to diagnostic classifications that are already known to the profession and deduces the type of case in particular (Shalem, forthcoming, p.7).

In that sense, the gap Muller sees between the theoretical and the practical refers to what must be bridged in the relation between the propositional and the practical if teachers are to be able to identify real events as instances of theoretical events, and make informed strategic decisions on how to act in the situation. It means that the teachers need a reservoir of academic knowledge – the propositional or 'knowing what' – to draw on, and it means that the link to the practical is made through inferences, which for both Muller and Shalem means that the teacher needs to 'know how' to make such inferences.

Within teacher education, this becomes very real in trying to develop assessment criteria for student teachers in their practice teaching, and the need to award a final mark to this teaching practice. As Rusznyak discusses, to base the assessment entirely on what the students do is not sufficient; if student teachers are not able to explain why they make the decisions they do and reflect on them in substantial ways, they are not empowered professionals who can make inferences on which to base professional judgements. Thus, teacher educators also need to engage with the extent to which student teachers draw on the established body of knowledge in diagnosing and 'treating' problems in the classroom – in other words, to what extent they have the propositional knowledge and can close the discursive gap to practice sufficiently well.

Rusznayak draws on Shulman's important distinction between pedagogical reasoning and pedagogical practice, and describes the development of an assessment rubric that maps both the cognitive dimension of teaching, and the observable classroom performance. It is not enough that student teachers reflect on their teaching, that reflection must be informed – it must relate back to knowledge for, in and of teaching.

These issues obviously also apply to pedagogical, context and curriculum knowledge. Knowing that learners from less privileged backgrounds have problems decoding invisible pedagogies does not make it easy to adjust one's teaching accordingly. Having a sense of the possibility of a particular disciplinary domain in relation to the educational task (including the development of citizenry), and understanding the different paradigms or philosophies of the discipline as it has evolved in history, may mean reading the curriculum differently, but it still needs to be recontextualised and operationalised to be related to the learners' prerequisites and implemented in 45 minute lessons.

The links between the knowledge domains will also play out differently in different contexts. A recent study in Denmark (Lindenskov, 2012) developed materials for supplementary mathematics teaching. The materials were inspired by materials from elsewhere but had to be adopted to the Danish situation, where learners' motivation and interest are considered central, where teachers must know how to involve parents in the learning work, where the learners are not told but guided through exploring, and where the material was more about directing the teachers to what to explore in order to respond to the learners better, than about how to present. This is clearly a *knowledge-of-practice* perspective, but again we want to point to how this is informed reflection; that is why a set of materials developed by specialists is used to direct the teachers' attention to the relevant elements of the teaching-learning situation. This direction of their attention is aiming at developing a trained gaze which is strongly anchored in specialist knowledge about how children learn mathematics (see also Schifter, 1998).

A Danish teacher trained in this tradition would battle to adapt to the South African classroom, and *vice versa*. Yet we need to be able to describe the knowledge, practices and gazes/stances of teachers in both contexts using the same concepts. In our view, we can only do so through focusing on the relations between the knowledge domains and the three aspects of teacher learning. This is supported by studies such as one by Ainley and Luntley,

where they found that experienced teachers draw on what the authors call ‘attention-dependent knowledge’, paying attention not only to the content of learners’ statements but also to the intentions of these statements (Ainley and Luntley, 2007).

## Teacher education and teacher learning

There are three papers in this issue which engage the content of teacher education programmes and teacher learning specifically. It seems only logical that teacher education must have strong relations to the different aspects and domains of teacher learning. And if we are right, the relations between the aspects would also need to be engaged. For instance, the substantive or propositional within curricular knowledge can address the current curriculum as well as the principled curriculum, but linking it to the practical knowledge aspect would be where issues of how to select and sequence content comes into play (something many mathematics teachers in the Western Cape were seen to struggle with (Reeves and McAuliffe, 2012). This also shows that there are also two dimensions to the practical dimension of curricular knowledge, namely how to implement the curriculum and how to critically engage it.

Taking a more general and macro view of knowledge domains, Sosibo focuses on the kinds of knowledge domains that are prioritised in a commerce stream of a B.Ed programme. The Minimum Requirements for Teacher Education Qualifications framework stipulates five teacher knowledge domains, namely, disciplinary, general pedagogical, practical, fundamental and situational knowledge, and Sosibo uses these to cluster the data that emerged from interviews with students and teacher educators on this programme. It emerges that the programme places greatest emphasis on general pedagogical and practical knowledge, and the least emphasis on fundamental and situational knowledge. This was of concern for students who felt that they were not prepared to teach in under-resourced schools, again pointing to the gap between propositional and procedural knowledge in the various domains.

Christiansen also engages a formal teacher education programme, and considers the extent to which a PGCE programme prepares mathematics teachers to teach effectively in the local context. A key question underpinning the paper is to what extent teacher knowledge and competence have an impact on learner achievement, given the overwhelming influence of the socio-

economic and home background. The paper presents an analysis of the assessment tasks of the PGCE for maths teachers, using three criteria: what knowledge domain do the tasks assess; do the tasks emphasise a knowledge code (that is, *what* you know, is important) or a knower code (*who* you are, is important); and to what extent the tasks focus on contextualised or decontextualised knowledge. She found that the programme tended to focus on the application of decontextualised content. The latter links to our concerns about being able to infer from academic knowledge to diagnostic knowledge; perhaps what is necessary is to have what Maton refers to as a semantic wave Maton (in press), moving between contextualised and decontextualised, and in the process demonstrating ways of bridging the discursive gap between theory and practice?

Interestingly, only one paper in the issue addresses the specifics of teachers developing their knowledge, practices and gazes. Bansilal's paper foregrounds the knowledge that is acquired and generated by teachers when they take an inquiry stance. She uses the narratives of four mathematics teachers who enrolled for a master's degree to show that these teacher-researchers were able to develop their knowledge for mathematics teaching as a result of their classroom inquiry. The teachers develop both propositional and practical knowledge, and the process of engaging with a systematic classroom-based inquiry seems to help them to bridge the discursive gap described earlier. This is the only paper in the volume that points to a possible process of *how* teachers may do this, and we see parallels to another South African study of how mathematics teachers learn through challenges in a context of solidarity (Brodie and Shalem, 2011).

## Specialised knowledge in teacher education

Above, we have discussed what we consider different aspects and domains of teacher knowledge, and claimed that it is the relations between these that teacher knowledge comes to life, so to speak, in terms of making *informed* professional judgments or inferences through drawing on specialised knowledge. The question is, does such specialised knowledge exist? It does, as also highlighted by Shalem, who convincingly argues that both condensing a case and characterising it, both diagnosing a problem and treating it, should draw on specialised knowledge. But the extent and coherence of this body of knowledge appears to vary from discipline to discipline. Concepts are not clearly defined in relation to the empirical (weak grammaticality), making it

necessary to reiterate concepts in every study, and concepts are often not clearly related, making it difficult to determine when theories are redundant or what new a concept adds to the field. Variations in the extent to which bodies of specialised knowledge were reflected in the submissions to this special edition on teacher knowledge illustrate this.

There were twenty-one submissions, which seems to indicate a healthy interest in the field. Eight of these were in the field of mathematics education, and one in science education, while the others were more generally in the field of teacher education. This may be an indication that in South Africa at present, as worldwide, the field of mathematics and science education is more strongly focusing their research in the area of teacher knowledge, and is developing a more precise language of description to do this research. There were no submissions on teacher knowledge in the field of literacy and teaching reading, or within the social science and humanity subjects. Yet the work by Christie analysed by Shalem indicates that it is not only possible but constructive to develop specialised knowledge for the teaching of English, for instance (Christie and Macken-Horarik, 2011). And some of Maton's work may have the potential to provide concepts which can be given specialised interpretations – for instance, the work by Adler and colleagues distinguishing teacher education lectures on the basis of the legitimising appeals to specialised mathematics education knowledge or the lecturers' personal experience (Parker and Adler, 2012), could be seen as distinguishing between stressing the epistemic relation versus stressing the social relation (Maton, 2007).

This distinction was also reflected in a meta-level study, comparing teacher education programmes across four countries (Rasmussen and Bayer, 2011). Rasmussen and Bayer found no major differences in the programmes between countries with high versus low performing learners, but they did find some differences in what types of knowledge is foregrounded. Interesting to us is the distinction they made between knowledge with a scientific versus an empirical basis. Perhaps we can now think of teacher knowledge as a double storey house with an attic – teacher knowledge on one level, teacher education on another but matching the rooms underneath, and the attic a space where issues of how to measure teacher knowledge and learning and their relations to learners' achievements and attitudes are engaged. In that sense, many of the papers in this issue span several levels; for instance, the papers by Rusznyak, Adler and Patahuddin, and Krauss and Blum all engage issues of measurement which have to relate to types of teacher knowledge in order to achieve their

purpose. Or perhaps teacher knowledge is like a fairy tale house with secret passageways and mysterious staircases, which is there as a physical entity (propositional), in which people do things (procedural) and which is also a lived-in space with emotional associations (personal). Our different aspects and domains are windows into the house, yet to describe the house better, we have to be aware of how the rooms relate and how best to get from one place to the other. Teacher education may then be seen as the blue print or the plumbing and wiring of the house . . . the metaphor is there to be played with.

A couple of recent works have shown the limitations of using one language of description only to try to distinguish practices, and have promoted combining descriptions of pedagogy with descriptions of relations to knowledge. Parker and Adler (2012) shows that when looking at the type of activity in the classroom, two lessons may appear similar, but their legitimation codes reveal just how different they are in their relation to knowledge. Naidoo (2012) shows that distinguishing between visible and invisible pedagogies does not capture the conceptual depth of the lesson, for which she instead draws on the systemic functional linguistic concept of co-extensions of meaning. We welcome this increased focus on relations to knowledge combined with other aspects and domains. However, they do not yet form a more coherent perspective on teacher knowledge, practice and learning. For the languages of description in teacher education to gain stronger grammaticality and verticality, we have to find ways to describe teacher education which can span a range of contexts, and which adds to our understanding of the ways of bridging the gaps between ‘knowing what,’ ‘knowing how,’ and having the gaze of ‘being’ a teacher.

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# Recontextualising items that measure mathematical knowledge for teaching into scenario based interviews: an investigation

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## Abstract

This paper interrogates the recontextualisation of available assessment items developed for research purposes that measure mathematical knowledge for teaching, into scenarios for use in qualitative studies related to mathematics teachers' subject matter knowledge. It draws from interviews with teacher participants in the Wits Maths Connect-Secondary project and their responses to two selected items from the Learning Mathematics for Teaching (LMT) project. The analysis shows that carefully constructed multiple choice items in the domain of (mathematics) subject matter knowledge have much potential in provoking teachers' talk and their mathematical reasoning in relation to practice-based scenarios; and exploring with teachers a range of connected knowledge related to the teaching of a particular concept or topic. We argue that productive use of such items further requires that researcher make explicit the mathematical ideas they expect to explore and assess in the developed items.

## Introduction

In their comprehensive survey of assessing teachers' mathematical knowledge for teaching (MKT), Hill, Sleep, Lewis and Ball (2007) make a useful distinction between "*the quality of mathematics instruction*" (which has embedded in it, value judgments on instructional approaches); and "*the quality of mathematics in instruction*" (p.150), where focus is "specifically on the actual mathematics deployed in the course of a lesson". Mathematics in use is thus professional knowledge, deployed for the purposes of teaching, and not for its own sake. What then is entailed in accessing and assessing such knowledge? Hill *et al.* (2007) describe the qualitative research, based on

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observation of teaching (knowledge in action), and/or task-based interviews where tasks are developed from knowledge of practice, and their more recent quantitative research in the Learning Mathematics for Teaching (LMT) project which has focused on developing, validating and then using measures of professional knowledge. Despite convergence in appreciation for the multi-faceted and complex nature of professional knowledge, these two lines of research remain disjoint. Hill *et al.* (2007) suggest that “. . . qualitative researchers have much to learn from large scale test developers” and vice versa, and that “. . . more cross over needs to occur” (p.152).

The work we report here moves into the terrain of crossing over. In the Wits Maths Connect-Secondary (WMC-S) project we are concerned with teachers’ mathematical knowledge in use, and its growth through participation in professional development and over time. To this end, we have selected LMT items, and recontextualised them into a semi-structured interview setting. In this paper we interrogate our use of two such items in depth to explore the potential of such recontextualisation for producing what we see as ‘fit for purpose’ readings of project teachers’ professional knowledge.

We begin with a selective review of the literature base on assessment of mathematical knowledge for teaching, followed by a description of the research project, and our research and development approach to professional knowledge. We then describe the interviews and the selected items and present our analysis of the teacher interview data. Our analysis will show that carefully constructed multiple choice items in the domain of subject matter knowledge (see below) have much potential in (1) provoking teachers’ talk, and their mathematical reasoning in relation to practice-based scenarios; and (2) exploring with teachers a range of connected knowledge related to the teaching of a particular concept or topic. In addition, from a methodological point of view, we will argue that productive use of such items in semi-structured interviews requires researchers to make explicit their assumptions as to what knowledge(s) are privileged in their assessments. Facilitating Hill *et al.*’s call for greater cross over and accumulation in our research, rests particularly on this latter point.

## Assessing teachers’ professional knowledge - what has been done?

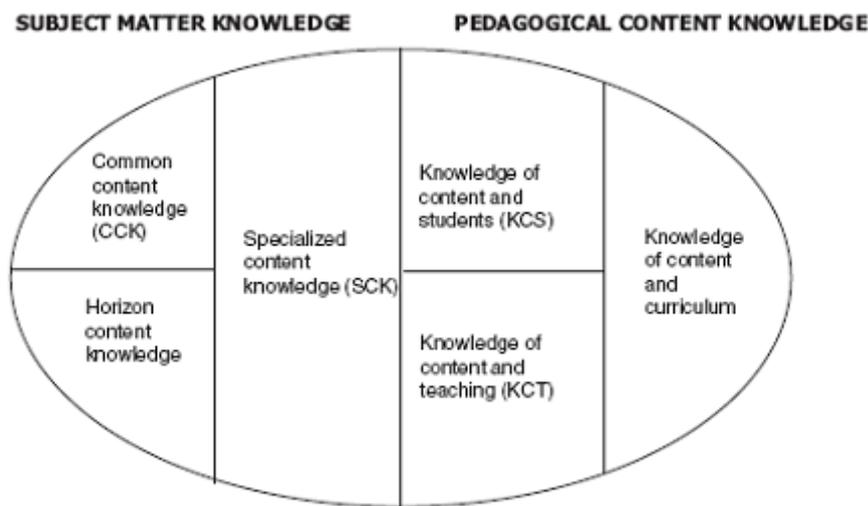
Research related to mathematics teachers’ professional knowledge (MKT), what it is, its relationship to practice and learning gains, how it grows, and

more recently, how it can be validly and reliably measured, has mushroomed. A comprehensive review of research on assessing MKT in the US, focused on “what knowledge matters and what evidence counts”, traces the development of methods for describing and measuring *professionally situated* mathematical knowledge in the US (Hill *et al.*, 2007). Briefly, in the 1980s and 1990s methods were geared towards uncovering mathematical knowledge for teaching through observations of teaching practice (e.g. Leinhardt and Smith, 1985), and/or exploring and describing teacher knowledge in task based interviews (e.g. Ma, 1999; Borko, Eisenhart, Brown, Underhill, Jones and Agard, 1992). Through studies of expert mathematics teachers, experienced teachers across cultural contexts, and of the complexity of learning to teach respectively, this work has contributed significantly to elaborating the specificity of professional knowledge in and for mathematics teaching. Hill *et al.* locate the recent measures work, and their LMT project, in the context of this qualitative research. They argue that, notwithstanding its advances, a major weakness is that it is necessarily small scale. They build from this work to enable large scale, reliable and valid ways of assessing professionally situated knowledge.

The results of the LMT research have been widely published and include reflection on how, building from Shulman’s (1986) initial work, the development of measures simultaneously produced an elaboration of the construct MKT and its component parts. As they developed measures, they were able to distinguish and describe Subject Matter Knowledge (SMK) and Pedagogic Content Knowledge (PCK), and categories of knowledge within each of these domains as illustrated in Figure 1. Common Content Knowledge (CCK – mathematics that might be used across a range of practices) was delineated from Specialised Content Knowledge (SCK – mathematics used specifically in carrying out tasks of teaching) (Ball, Thames and Phelps, 2008). Simply, recognising an incorrect answer to a calculation (CCK) is not synonymous with being able to reason across a range of responses to a calculation as to their mathematical validity and worth, as task teachers continuously do (SCK). Within PCK, where knowledge of mathematics is intertwined with knowledge of teaching and learning, they distinguish Knowledge of Content and Students (KCS – e.g. knowledge about typical errors learners make, or misconceptions they might hold), from Knowledge of Content and Teaching (KCT – e.g. knowledge of particular tasks that could be used to introduce a topic). All their items are presented in multiple choice format, and whether SMK or PCK, are set in a teaching context. In addition to describing their MKT constructs and exemplifying measures of these, they have elaborated the work done to produce their measures (Hill, Schilling and

Ball, 2004), and reported on positive correlations they found in their study of the relationship between measures of teachers' MKT, the quality of their mathematics teaching and their learners' performance (Hill, Rowan and Ball, 2005; Hill, 2008).

**Figure 1: Domains of mathematical knowledge for teaching (MKT)<sup>3</sup> (Ball *et al.*, 2008)**



In their concern for construct validation, the LMT project has subjected its work to extensive critique. A whole issue of *Measurement* (Vol. 5, No.2–3, 2007) is turned to this purpose, and makes visible just how complex, and costly, such assessment practices are. Invited commentary highlights the limitations of quantitative measurement of professional knowledge. Schoenfeld (2007, same issue) argues that any assessment must be explicit and clear in what is being assessed, and that it is not clear what exactly individual LMT items and their distractors do, nor how they accumulate. Difficulties entailed in measures work are critiqued within the LMT project itself, particularly PCK items aimed at KCS (Hill *et al.*, 2007; Hill, 2008). The strength of the construct of PCK, in their terms, depends on how well it can be distinguished from knowledge of the mathematical content itself. LMT validity tests, including clinical interviews on these items, failed to separate

<sup>3</sup> We include the figure of Ball *et al.*'s components of MKT to assist the reading of our paper, as we refer to these through their abbreviations here and later in the paper; it is not as an object of attention in itself.

KCS from related measures of content knowledge. Scores on KCS items correlated highly with CCK scores. As Alonzo (2007, same issue) comments, this result has the danger of suggesting that all that matters is content knowledge, back-grounding the important work and progress that has been made across the field, in elaborating professional knowledge and particularly SCK.

Hill *et al.* (2007) and Hill (2008) describe additional insights from their cognitive interviews on PCK-KCS items that showed that teachers also used mathematical reasoning, and test-taking skills, to decide on the correct answer. Teachers were asked to ‘think aloud’ as they talked about each item, their selection of the correct answer from four possible answers in the multiple choice format, and their justification for their selection. In their analysis of teachers’ talk, it was difficult to separate out teachers’ KCS from their mathematical reasoning about their choices, and so their SMK in use. Hill *et al.* (2007) conclude that ‘this domain [PCK] remains underconceptualised and understudied’ (p.395), despite wide agreement in the field that this kind of knowledge matters. Their reflection on their detailed PCK work presents considerable challenges for the field of mathematics education: the notion of PCK is widely invoked in mathematics teacher education research and practice, often without clear and operationalised definitions.<sup>4</sup>

The nature of the boundary between SMK and PCK has been critiqued by others researching in mathematics education. Huillet (2009), for example, argues from the perspective of the Anthropological Theory of Didactics that there is no ‘common content knowledge’; all knowledge is tied to activity. Hence, clear distinctions between SMK and PCK are problematic. Much of the critique on a hard boundary, including Huillet, has emerged from qualitative studies with stronger situative perspectives, and focused at the secondary level (e.g. Zazkis and Leikin, 2010; Nardi, Biza and Zachariades, 2012). WMC-S also has a strong situative perspective, but nevertheless took up the challenges of ‘crossing-over’ and using LMT measures. Interestingly, despite our selections of LMT items that assess SCK, we too will raise questions about the boundary between SMK and PCK in teachers’ mathematical reasoning. Construct delineation and validation is a strong feature of quantitative research, and central to the work of Krauss, Baumert and Blum (2008) in their large scale study of secondary mathematics teachers’ professional knowledge

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<sup>4</sup> Nardi, Biza and Zachariades (2012)’s study of teachers’ argumentation in task-based interviews contributes to this debate. It provides a careful operationalising of forms of argument, and teachers’ practical, yet complex reasoning.

and its relationship to learner performance. Based in Germany, their measure development and use in the COACTIV<sup>5</sup> project, like Hill *et al.*, worked from the assumption that professional knowledge is situated, specialised, and thus requires assessments that are not synonymous with tests at particular levels of institutionalised mathematics (be this school or university). Indeed, for Krauss Baumert and Blum, secondary teachers' SMK (what they call Content Knowledge – or CK) sits in a space between school mathematics and tertiary mathematics (p.876), and is clearly bounded from their interpretation of PCK. They report on two hypotheses related to *professional knowledge* and *growing knowledge*. They conducted CK and PCK tests on different groups selected with respect to professional knowledge (i.e. mathematical knowledge in and for teaching): two groups of experienced secondary mathematics teachers with different mathematics pre-service training; other teachers (of biology and chemistry); mathematics majors in university; and students in their final 13th year of mathematics study in school. Their results confirmed their professional knowledge hypothesis – experienced teachers irrespective of their teacher education route showed high PCK scores.

At the same time, however, mathematics major students performed unexpectedly well on PCK items. Krauss, Baumert and Blum (2008, p.885) explore this interesting outcome in their study – how it was that mathematics major students, who had no teaching training or experience, were relatively strong on their PCK items. We zoom in here to bring into focus the diverse ways in which professional knowledge constructs have been operationalised in the field. For example, some of the examples of Krauss *et al.*'s PCK items are more aligned with Ball, Thames and Phelps construct of SCK, than with their elaboration of PCK. Although both research groups include knowledge of students, and knowledge of tasks as PCK, their interpretation of these into measures differs. Krauss, Baumert and Blum, for example, exemplify a PCK task item that asks: “*How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem as possible*”. The sample response given includes both *an algebraic and geometric representation* (p.889). In Ball, Thames and Phelps' terms, this response does not require specific or local knowledge of students, nor of curricula, or particular teaching tasks, and hence, in their terms would be SCK, and distinct from PCK. In other words, knowledge of multiple representations shifts between PCK and SMK across

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<sup>5</sup> COACTIV refers to the project on Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Student' Mathematical Literacy.

these two studies. We do not go further here into other studies that have developed measures of SMK and PCK e.g. TEDS as reported in Tatto and Senk (2011), and MT21 reported in Schmidt, Blömeke and Tatto (2011). While they are in themselves of interest, they do not add to the key issue that emerges from a review of measures research in MKT: this construct and its components are differently operationalised in different studies, a point made by Hill *et al.*, (2007) and noted as a shortcoming in this research.

A major reason for this incoherence refers us back to Schoenfeld's comment that it is necessary that the responses expected on assessment items are made explicit. Descriptions of the principles guiding construct delineation on their own are insufficient. Specifically, what needs to be made clear are the kinds of knowledge and reasoning, be it mathematical or pedagogical, that are being provoked, at least at the level of intention, in and across items, and thus assessed. Readers of quantitative research are not privy to the conceptualising processes behind items exemplified. Even though COACTIV uses open ended and not multiple choice items, as a quantitative study we do not see the mathematical analysis behind the items, nor the detail of the analysis and coding of teachers' responses. For this would be to reveal the items, and thus muddy any ongoing research related to the measures. We have done our own mathematical analysis of the items we use, which we will argue is a critical step in their recontextualisation. Whether this matches the underlying analysis in their construction is not relevant. However, as most items are confidential, and as will become visible later in the paper, there are constraints on our reporting, and so on developmental cross-over.

## The WMC-S research and its use of LMT items

The two items we describe, analyse and then discuss below were used in a semi-structured interview with 30 teachers participating in the WMC-S project. Both were identified as SMK in LMT and in algebra (a content focus in the project), one on linear equations and one on quadratics. Before we proceed with the detail of the methodology we used, we provide a brief introduction to the project, and our orientation to teachers' professional knowledge and how this can be read within an interview setting.

WMC-S is a 5-year research-informed and data driven development and research project working with the mathematics teachers in ten schools in one district in Johannesburg, South Africa. Mathematics in use in teaching (i.e. *in*

instruction) is a central focus of WMC-S. Its vision and intervention model has been shaped by previous research on teacher development (Adler and Reed, 2002) and the follow-on work of the QUANTUM project (Adler and Davis, 2006; Adler and Davis, 2011), with its focus on mathematical knowledge in and for teaching. WMC-S professional development (PD) work thus aims to enhance teachers' mathematical knowledge for teaching. Our practice is guided by deliberate teaching focused on key mathematical objects of learning (Marton, Runesson and Tsui, 2004), and thus with a bias towards SMK as elaborated by Ball, Thames and Phelps (2008) (i.e. CCK and SMK), or towards CK and PCK in Krauss, Baumert and Blum's terms; and we are researching this process together with the maths teachers in participating schools.

Our structuring of the scenarios in the interviews is a function of our orientation to the centrality of evaluation in pedagogic communication (Bernstein, 2000). For Bernstein, pedagogic communication condenses in evaluation (p.36). While communication about practice in an interview is not synonymous with a pedagogic encounter between a transmitter and acquirer, it nevertheless provokes discussion of school mathematical knowledge, and with this legitimating criteria as to what counts as appropriate knowledge in the interview context. As previously argued, teachers call in a range of knowledge resources in their teaching through which we can read how criteria come to work, and so what knowledge is made available to learn. Similarly, what teachers recruit to legitimate what counts as appropriate knowledge in scenario based discussion in an interview setting can be read as their knowledge in use.

In this context, the LMT items appeared useful precisely because of their multiple choice format, and the way in which we used these. We asked teachers to consider the scenario and then discuss with us, each of the four multiple choice options offered. Based on our engagement with LMT items in a training workshop the previous year,<sup>6</sup> it seemed productive and efficient to use available items in our interviews. As interview items, they were necessarily recontextualised. In addition to situational change, we did not ask teachers to choose an answer and then, thinking aloud, justify their choice. Nor did we present a scenario, and ask for open comment. We gave the scenarios together with the multiple choice responses to teachers two weeks before the interview, and asked teachers to prepare for the interview by reading the scenario and considering each of its four options. In the interview,

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We are indebted to the generosity of LMT, and the time and resources committed to sharing the LMT knowledge and experience with WMC-S. Their workshop at Wits, December 2009, enabled the work reported here.

we would then ask them to share their thinking about each option within each scenario with us. Our purposes here were to set up a context within which teachers would be required to reason, and talk about connected mathematical ideas. We would then be able to develop a dynamic reading of mathematical knowledge for teaching in use in the interviews within and across all the teachers. This recontextualised use marks out its difference from the clinical interviews done for validation purposes in LMT on the one hand, and task based interviews in similar qualitative teacher interviews on the other.

LMT has released a set of items for public use. However, the two SMK items that we discuss here were drawn from the full set of items to which we had access for use in the project, but which remain confidential. As we hope will become evident, the items we chose have been very productive in eliciting teachers' talk and so information from which we could read their knowledge in use. However, as with all other reports on LMT item work, we are required to mask the detail of the items here. This inevitably constrains our reporting. As each item was set in a teaching context, our descriptions below include a description of this context or scenario, followed by the multiple choices. We present Scenario 1 in detail: its mathematical analysis, coded teacher data and discussion of three teachers' responses. Space limitations necessitate a brief treatment of Scenario 2.

## Scenario 1

### **Mathematical analysis:**

*Scenario 1 presents an interaction between Grade 10 learners on solving an equation in the form  $ax^2 = bx$ ,  $a, b$  natural numbers, and  $b$  divisible by  $a$ . The discussion is focused on why, if you can divide both sides by  $a$ , can you not also divide through by  $x$  to get  $x = b/a$  (also a natural number) as the solution?*

*After the presentation of this situation, the LTM item asks respondents to select the most appropriate response from four learner responses that followed in four bulleted points, each with some reasoning for or against this solution method offered. These related to (1)  $x$  being a variable, i.e. divisibility by  $x$  is not allowed because it varies (2)  $x$  being a real number, (3) finding the square root i.e. the solution requires finding square roots, and (4) composite reasoning involving dividing by 0, and the quadratic form of the equation.*

As already noted, we changed the requirement for selecting the most

appropriate response, asking instead that teachers consider and then discuss with us, their interpretations of each of the four student responses. We were interested in using this item in our interviews to see how teachers engaged with each of the four responses offered, and so with the connected and inter-related concepts embedded in this scenario; specifically, what we would regard as subject matter knowledge related to solving quadratic equations.

Mathematical analysis of the item as a whole involved unpacking the general form of a quadratic equation, its structure, roots, together with attention to those features reflected in the multiple choice responses of statements made about the solution strategy. Through this analysis we identified and then coded seven such connected concepts or mathematical ideas embedded in the scenario: (1) the notion of the variable  $x$  in the equation (Mv), and awareness that division by a variable could include  $x = 0$  on the one hand and thus a constraint on  $x$  would be required, and that while  $x$  was a variable, its value was *an* unknown (2) the real number system (MR) with respect to possible solutions and awareness that the fact of  $x$  being a real number was not an argument for its divisibility, and again the constraint of  $x = 0$ ; (3) the quadratic equation and its two solutions (MQ), and thus that obtaining one solution indicated that the solution was incomplete; (4) non-divisibility of 0 (MΦ); (5) understanding of square root i.e. its meaning and conditions (MSq); (6) demonstration of a method for solving a quadratic equation (MDm); and (7) the understanding of the logic of a composite statement related to division by zero, and thus the ‘loss’ of one solution (MLg).

### *Coding the interview data*

After a first careful and systematic analysis of all teachers’ talk on this item in the interview, and using the coding above, we found this initial analysis masked the quality of responses, and produced a reading of absences within and across teachers. We thus refined our coding by adding labels positive, zero, and negative to each code. For example, MQ+ means the response clearly indicates recognition of a quadratic equation having two solutions. We used MQ<sup>0</sup> to indicate that there was insufficient data to claim whether or not there was such recognition, and MQ- when the teachers’ talk contained incorrect mathematics. Simply, positive (+) indicates correct; zero (0) we cannot claim anything with respect to teachers’ SMK with respect to a particular concept; and negative (-) indicates incorrect or a misunderstanding. We tested and then applied this coding across the full set of interview text related to this scenario.

## Results of our interview analysis: Scenario 1

In the Appendix, we present a summary of our analysis against each of the seven key mathematical ideas discussed above for all the teachers.<sup>7</sup> This result of analysis shows clearly that whilst the scenario potentially provokes a wide range of mathematical discussion, such discussion was not present across all teachers. Teacher *T10*, for example, confidently discussed the quadratic form of the equation, and demonstrated a method to ensure two solutions were obtained. But, he did not engage with  $x$  as real variable, hence the 0 coding for both ‘real’ and ‘variable’. This absence of discussion cannot be taken as an absence of knowing.

We move on to present three cases in detail, Teacher 19, Teacher 10, and Teacher 3, each of whom had different qualifications and years of experience, but all are qualified secondary mathematics teachers. While their histories and contexts of teaching are significant in the programme, these are not relevant to our purposes here. We have selected these three teachers because together they illuminate the variation of teachers’ responses on identified key mathematical ideas on the one hand, and how these emerged within the social setting of the interview on the other.

The responses of the selected three teachers are summarised in Table 1 below, and the coding in the table is used in the analysis of the interview extracts that follow.

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<sup>7</sup> The table presents coding for 24 teachers. At the time we did the coding (2011), five teachers interviewed in 2010 had moved schools and 1 teacher did not give any answer on this scenario. We excluded them from our analysis, as they will not be tracked.



In analysing the interview, we saw potential in the above coding for enabling us to track teachers' professional knowledge related to this item. We were also interested in the dynamics of the teachers' mathematical reasoning, and the knowledge resources called in and so how the item played out in the interview setting. Specifically, we were interested in whether teachers' responses drew on their SMK on the one hand, and how responses were shaped by the way the interviewer presented and then probed in relation to the scenario. In the detailed analysis below we present the full extract from each of the three teachers' interviews, followed by a brief analysis that connects to the relevant coding categories and highlights key features of the interviews.

Teacher 19 had, as requested, looked at the scenarios before the interview, and thus was prepared for the discussion about them.

**Table 2: Teacher 19 interview and its coding**

| Interview 19   | Inter-viewer                    | Teacher's knowledge  |
|--|---------------------------------|--|
| <p>I : So there's this little episode in class and then here are some statements. <u>What do you think about these statements?</u></p> <p>B : You know with the first statement <u>I think the learner does not understand the, doesn't know the difference between linear and the quadratic so that</u> is why here he divide everything, he divided everything by a which is fine, right, and then now he got this. <u>This is a quadratic, but then the learner cannot see that it's a quadratic. He divided by <math>x</math>, right, whereas we are looking for <math>x</math>. So he divided by <math>x</math> and then it means he lose what - he lose one value of <math>x</math>. So he lost that and then he left with 1 as if it was a linear, . . .</u></p>  | IGP                             | <p>PCK – KCS</p> <p>MQ – recognises the quadratic with 2 solutions (implicit)</p>  |
| <p>I : OK</p> <p>B : So it means <u>the learner does not understand the difference between the quadratic and the linear.</u></p>   | IGP                             |  |
| <p>I : OK. <u>So what do you think about these statements?</u></p> <p>B : You see these statements I read and then I couldn't understand. OK, <i>[Read Bullet 1]</i>. So here if since <math>x</math> is a variable it can differ. I don't know, I mean yes it can differ because I mean <math>x</math> is a letter, right? <u>Obviously because it's a quadratic you are going to have two different answers.</u></p> <p>I : Right</p> <p>B : So here I don't understand also... So <i>[Re-read Bullet 1]</i>, so of course you can't do different. I don't understand that. <u>Actually I didn't understand that.</u></p>  | IGP                             | <p>MV+; MQ – States she does not understand option 1, reads it, recognises <math>x</math> as variable, then states quadratic has 2 solutions (explicit).</p>                         |
| <p>I : I think what they mean here, and that would be interesting to hear what you think, is that what this child is saying is that the reason why you can't cancel <math>x</math> here and <math>x</math> there is because here <math>x</math> might be 3 and here <math>x</math> might be 2.</p> <p>B : Oh, OK</p> <p>I : <u>I mean is that a right kind of reasoning?</u></p> <p>B : No, no, no. I don't know it's not, it's not.</p> <p>...</p> <p>B : Ja, but like he says, what did the learner want to say here? He tried to say you can cancel <math>x</math> here on both sides but then, but because the <math>x</math> can vary, you cannot do that. That's what you're saying. But I don't know. <u>Again I will just insist on saying that it's because you know what, they just have to know the difference between quadratic and the linear equation.</u></p> | IC – IPV interprets for teacher | <p>MV0 – recognises flawed reasoning, but asserted not justified</p> <p>MQ+ – dominant justification</p>   |
| <p>I : OK, alright, excellent.</p> <p>I : <u>So what about this one? <i>[Read Bullet 2]</i> You agree with that?</u></p> <p>B : But then you don't know yet. You just know that it's just... You don't know yet that it's a real number or it's going to be a fraction.</p> <p>...</p> <p>B : Ja, so it cannot be a reason.</p> <p>I : OK. <i>[Read Bullet 3]?</i></p> <p>...</p>  | IPR                             | <p>MR0 – Recognition that the solution could be non real again asserted not justified. (where important idea here is that <math>x</math> could be <math>0 \in \mathbb{R}</math>)</p> |
| <p>B : Like most of the time when they were, OK, OK, where did you, when you have something like that you have to get rid of two.</p>  | IPSq                            | <p>MSq+ – Recognise finding the square root is not leading to</p>  |

|  |   |   |
|--|---|---|
| <p>I : Right</p> <p>B : So if... Still the problem is that if it's not that .... because once you do that you can see <math>x</math> is there and <math>x</math> is also there. <u>So if you find the square root of ... here you will find the value of <math>x</math>, but then here as well you will find the <math>x</math>, so what are you actually looking for?</u></p> <p>I : Right. It's not helping.</p> <p>B : So it's not helping.</p> <p>I : OK. <u>And the last one?</u></p> <p>B : The last one, [Read Bullet 4]. Yes I can. . . <u>You know what, when I look at this, because these people they don't understand the difference between</u></p> <p>B + I: <u>a linear and a quadratic</u></p> <p>I : so none of these really help</p> | <p>I-revoices</p> <p>IP<math>\Phi</math>, R,<br/>Lg</p> | <p>solution</p> <p>PCK-KCS; MQ+; Lg-Quadratic recognition dominates her reasoning; she did not engage with the composite argument (Lg) in Bullet 4.</p> |
|--|---|---|

**Coding: Interviewer utterances**

IGP – **General probe** – interview asks what the teacher thinks about each/all statements;  
 IPV, IPSq, IPR, IP $\Phi$  . . . **specific probes**, following the seven key concepts and codes identified above.  
 IC – **Clarification** related to the scenario

What is clear from this interview extract is that the interviewer deliberately led the teacher to discuss each of the four multiple choice statements. For this teacher, we coded three of the key ideas as positive (the notion of variable, recognition of quadratic equation, and the notion of square root); and one negative i.e. her difficulties in understanding the composite statement in Bullet 4. We thus see from Table 2 that even though the interviewer deliberately asked for response to each of the four statements, three aspects (divisibility by zero, Real number system, methods of solving) were coded as zero. This does not mean that this teacher did not know these, but she did not talk about them. The teacher repeats what appears to be her dominant response to this scenario – the importance of recognising that it is a quadratic equation.

The interview data here illuminates the recontextualising issues as the item is adapted from asking ‘which one is correct’ to ‘what is your view of each of these statements’. The change opens a space for the teacher to talk about their teaching practice. Teacher 19, as she goes through the four options, talks about her learners and what they do, and what she would emphasise in her teaching. As a result, the item which is identified in LMT as measuring SMK seems here to provoke reasoning and discussion about students and instruction, and so PCK. Indeed, it is her PCK, one could argue, that shapes the dominant response given, and her lack of attention to Bullet 4, which in the multiple choice setting is the correct answer. It involves composite reasoning involving the possibility of dividing by 0, and ‘losing’ a solution. It is possible to read her PCK focus as a

side-stepping or avoiding the mathematics being probed. It is also possible, given her reasoning in the interview, that in a test setting, and having to choose between the four options, Teacher 19 would choose option 4, having eliminated the others. Interestingly, a number of our teachers found the fourth option difficult to interpret and discuss in the interview setting. Thus, from a measurement perspective, the validity of this item comes into question.

What begins to emerge from the detailed data and analysis here is that what is produced as teachers’ knowledge in use in relation to this recontextualised LMT item is indeed dynamic and an interaction between the item itself, what the teacher recruits into the discussion, and what and how the interviewer probes the teacher’s responses.

**Table 3: Teacher 10 interview and its coding**

| Interview T10 (also prepared for interview)   | Inter-viewer | Teacher’s knowledge                                       |
|---|--------------|---|
| I : [reading the problem] <u>What are your views on those statements?</u><br>J : (laughs)   | IPG          |   |
| I : <u>Ok, let’s go with the first one. What do you think of the first bullet?</u>  | IPV          |   |
| J : [ <i>Read Bullet 1</i> ] Well, I think my approach to this one would be most of these things are not right. If you look at the question there, it’s times a, it’s a times a. Once it is this one here <u>it is important for learners to know this is a quadratic thing. How many answers? Ask your learners. “Two answers.”</u> Good, you must get two answers; your $x$ has got to have two, um, answers, real numbers there, whatever it is. So these learners who are saying: “We divide by $x$ what what what”... Yes I understand but that’s not the procedure. <u>The procedure is it’s a quadratic, ok. Because quadratic therefore transposes the <math>x</math>, the <math>bx</math> to the other side, equate it to zero, quadratic equations must be equal to zero. Do you have seen that quadratic equation?</u> “No, we don’t have seen.” Thereafter, ok, fine, what do you do next? “ <u>Look for a common factor.</u> ” |              | MQ+ Quadratic recognition<br><br>MDm – Methods of solving |
| I : But what do you think about that learner who says: [ <i>Read Bullet 1</i> ]. <u>What do you think of that?</u>  | IPV          |   |
| J : That learner yes is partial because the main idea is for this learner to get $x$ , alright?   |              |   |
| I : Mmm   |              |   |
| J : So he is saying: “No, why should I worry? If I eliminate one $x$ but I still going to remain with $x$ then I’m still, I’m still fine, because what is required is an $x$ ”. Do you see that?  |              | PCK – KCS   |
| I : Mmm   |              |   |
| J : So the argument of that learner there you don’t need to take it lightly, you see? But unless you go back to the mistakes of the learners, what are their errors?  |              |   |
| I : Mmm   |              | MQ+   |
| J : That one must emphasise it’s quadratic, it must be two answers, right?  | IPR -Partial |   |

|   |                    |   |
|---|--------------------|---|
| <p>I : <u>Ok, and if [Read a part of Bullet 2]?</u><br/>                 J : If (reads part of bullet 2)<br/>                 I : Mmm<br/>                 J : Right? You remain with <math>x</math> on the left.<br/>                 I : Mmm<br/>                 J : And no, and no ri... and no <math>x</math> on the right. So you've got only one <math>x</math>.<br/>                 I : Ok<br/>                 J : You get one answer.<br/>                 I : Ok<br/>                 J : No, two answers. <u>So that must be very clear to the learners. You eliminate one <math>x</math> there, fine; you remain with one <math>x</math>, isn't it? Fine. Because this learner has a very good reason to say: "Hey, I can eliminate the 2, right? Divide both sides by 2. Why not dividing by <math>x</math> both sides?"</u> So the one learner who says: "No, no, I'm not dividing by <math>2x</math> both sides." Do you understand?<br/>                 I : Mmm<br/>                 J : The procedure of that learner is not wrong. It's also correct. But it's quadratic, it must have two answers. That one we must make it very clear, because those are some of the things that we need to understand.<br/>                 I : <u>So what do you think about the fourth bullet then? [Read Bullet 4]</u><br/>                 J : <u>That's again. Take, um, taking our learners back to the real numbers and non-real numbers, we have dealt with that one there.</u><br/>                 I : Mmm<br/>                 J : <u>It is not permissible to divide by zero. Not permissible at all. That's a mathematical, a mathematical suicide. Don't ever do it, don't divide by zero.</u> The situation is undefined. Do it on your calculator. Divide by zero; error. Divide by zero; error. Why? Not allowed. You cannot do it.<br/>                 I : Mmm<br/>                 J : You cannot divide by zero.</p> | <p>IPΦ, R, Lg,</p> | <p>MQ+<br/><br/>                 PCK – reasoning about learners, BUT possibility of <math>x = 0</math> not considered.<br/><br/><br/><br/><br/><br/><br/><br/><br/><br/>                 M Φ+ – divisibility by 0</p> |
|---|--------------------|---|

We coded T10 as having three of the seven key ideas correct, three as zero, and one as negative. At the same time, while T10 and T19 reasoned in similar ways in their interviews, the interviewer probes were not consistent, and so different ideas were provoked for discussion. Table 3 shows that there was no explicit probing of the third option related to the notion of a square root and so this is absent in T10's interview. Also the second bullet or option was only partially probed. As a result, this teacher did not talk about the idea of finding the square root of each side, and therefore we coded zero for understanding of square root. In this context, while the teacher did talk about variable values for  $x$ , divisibility by zero and the quadratic equation, his reasoning was shaped not only by the interview probes or non-probes but also the opportunities opened up for him to talk about his experiences of learners and his preferred teaching methods in dealing with this particular problem. As a result, many of the mathematical ideas embedded in this item are coded as zero to indicate that we do not have data on these. As with Teacher 19, the fourth bullet proved to be

difficult for this teacher. However, this option opened up the opportunity for him to talk about the notion of divisibility by zero.

It is also interesting to note here that while both T10 and T19 emphasise the importance of recognition of the quadratic equation, they do so in different ways. T19 emphasised learners (mis)understanding; T10 emphasised how to teach. And again, what emerges is blurring of PCK and SMK in the teachers' reasoning. What is thus produced as knowledge in use is clearly co-constituted by the item itself, the interviewer probes, and what the teacher recruits.

Teacher 3 too had prepared, and her responses evidence weaker SMK.

**Teacher 3 interview and its coding**

**Table 4:**

| Interview T3   | Interviewer                   | Teacher's knowledge   |
|--|-------------------------------|---|
| <p>I : [Read the scenario] <u>What is your view of each of these statements?</u> The statements where the teachers asked the rest of the learners to say their views about what these other learners were debating about? <u>Maybe if you look at the first one, [Read Bullet 1.</u></p> <p>T : This one, no. over here we are looking at <math>x</math> as the common factor, as the real...<math>x</math> as the common factor, meaning that as a common factor, as a common variable. <u>So it is possible for them to divide throughout. So the first statement it's out. If I have <math>2x</math>... if I have this I can divide throughout by <math>x</math>.</u></p> <p>And then they say [Read Bullet 2]. You can say yes it's a real number <u>but then we don't know the value of that real number so yes it's a common factor, we can divide throughout the <math>x</math>.</u></p> <p>And then [Read Bullet 3]. But then if you take the square root of [], or you say by then, <u>the one thing that if you're working with square root, most of them, some of them it confuses them...some not most, some. It confuses them. So if they find... especially when they have to find the value of <math>x</math>, they can only. . .they love working with square root, squares and square roots, or cube and cube roots, but then when you have to find the value of <math>x</math> and you get the comma something something, it even looks wrong to them. ...</u></p> <p>Then they say [Read Bullet 4]. <u>Meaning that the value of <math>x</math> that you are dividing with might be zero. [Re-read Bullet 4]. It's possible. Unlike when you take the other side, when you transpose the [], then look for the common factor, yes, you'd find two solutions, that <math>x</math> is equal to [], equals to [], and <math>x</math> is equals to []. But then you divide both sides by <math>2x</math>, you only get one solution, [] ...</u></p> | <p>IGP</p>                    | <p>MV0</p> <p>MR0</p> <p>PCK – KCS<br/>Msq0</p> <p>Methods of solving – MDm</p> <p>Quadratic recognition – MQ</p> |
| <p>I : <u>So at the end of it all, what is your response to this learner number two who says, you cannot divide both sides by <math>x</math>?</u></p> <p>T : No, you can. You can divide both sides by <math>x</math>.</p>   | <p>IPV – IC clarification</p> | <p>Real Number-MR-</p>  |

Table 4 shows that there was far less probing in this interview – a function of the teacher responding to all four bulleted statements without interruption. At the end, the interviewer asked “at the end of it all, what is your response to this learner number two who says, you cannot divide both sides by  $x$ ?”, and so goes back to the second option but not bullets 3 or 4. This may be the most critical question for the interviewer, who possibly noticed the contradiction in the teacher’s responses: T3 realised that dividing by  $2x$  lead to only one solution but previously said that “we can divide throughout the  $x$ ”, an argument she commits to. So here only one of the 7 mathematical ideas – a demonstration of the procedure to solve the quadratic equation – is coded as correct. Interestingly this demonstration is offered as the teacher responds to the fourth option. Here too we don’t have access to the teachers’ thinking on the correctness of the last statement. We do have access to her view that division by a variable ‘on both sides’ is possible.

Compared to T10 and T19, T3 engages less with the elements embedded in the item, and the interview shows that only one aspect was coded positive even though it was not accurate. T3 has a clear idea about the procedure of solving this type of equation, but the connected set of ideas related to a quadratic equation is not evident.

## Elaborating our analysis briefly through Scenario 2

*Scenario 2 is in the form of a teacher asking her learners to construct a story that would be appropriately modelled by a linear equation of the form  $y = ax + b$ ,  $x = 1, 2, 3, \dots$ , where  $a$  and  $b$  were given. Four possible stories are provided, and in the interview we asked teachers to consider each of these and which would be appropriate for the given equation. The first two of the stories generate the wrong sequence and the next two options generate the right sequence. The stories are obscured for confidentiality and presented in a general form.*

- *Option one uses  $b$  as a starting point in the story and plays with the phrase ‘make twice as many’ to produce  $b, 2b, 2(2b), \dots, 2^n(b)$ .*
- *Option two also generates a different sequence, here using  $a$  as a starting point, to which  $b$  is added. This played with the words ‘each time’, to produce the sequence:  $a+b, 2(a+b), 3(a+b), 4(a+b) \dots n(a+b)$ .*

- *Option three is the appropriate story playing with the words ‘two more’, and generating the sequence  $b+2, (b+2)+2, ((b+2)+2)+2 \dots b + 2n, n = 1, 2, 3, \dots$  which is well-matched with the linear equation given.*
- *The story in option 4 generates a similar numerical sequence but requires a different model. The starting point of the story is the number ‘ $c$ ’, where  $c = a+b$ . So  $c \neq a$  and  $c \neq b$ . It used the words ‘two more each day’, generating a sequence:  $c, c+2, (c+2) +2, ((c+2)+2)+2 \dots$ , and thus the same pattern as option 3. However, the model for this is  $y = c + (x-1)2$ , which is equivalent with  $y = ax + b$ .*

Scenario 2 is also identified in LMT as measuring SMK. We have provided as much detail above as possible to reveal the power of this item in the construction of the distractors, and particularly in how language is used in the four stories. For example, while all four ask a similar question at the end of the story, there is a play on ‘two more’, ‘twice as many’, and ‘two more each day’ across the options, thus probing the use of the mathematics register.

Most of the teachers interviewed found this question difficult. This observation is based on our analysis across the interviews where there was less talk about the item and its options. In addition, some of the teachers stated their difficulty e.g. “these are hard”. Zooming in here on T3, T10 and T19, an additional observation is that, in contrast to Scenario 1, none of these three teachers recruited pedagogic knowledge (i.e. knowledge of teaching or of learners) to discuss the item. They were preoccupied in understanding the story problems, and their related sequences, and then discussing their views of each of the stories in relation to the given equation.

T10 and T19 talked through how they worked out the sequence generated by each story. This then led to them being able to identify the incorrect stories. Distinguishing between those that generated the same sequence was more difficult. T10, in fact, was one of the teachers who stated that this problem was difficult for him, and that he does not focus on “word problems” in his teaching. The greater difficulty with this item was evident in T3’s interview where we have no data about her thinking of the four story options. While her engagement in Scenario 1 was more limited than T10 and T19, she nevertheless had much to say. Here, in Scenario 2, however, it seems the teacher struggled and the interviewer did not probe further.

What is clear here is that the differential familiarity across Scenarios 1 and 2 provide different access to teachers SMK in the interview setting. This raises questions then about how more difficult items are dealt with in the interview setting.

## Discussion

We remind readers that our interest in this paper is not the teachers' knowledge *per se*. Our interest is in the potential of items, developed and validated for purposes of measurement and correlation with learner performance and teaching quality, for illuminating teachers' mathematical knowledge for teaching in an interview setting. We discussed how the use of the multiple choice format suited our project and orientation to teachers' knowledge in use. We are seeking a dynamic descriptive assessment of teacher professional knowledge; but more importantly, we have interrogated particular items we have used to critically reflect on what such recontextualised use *does*. What reading is produced about teachers' knowledge in use? How does this 'fit'<sup>8</sup> with our purposes and with what constraints?

The first point we wish to make is that, despite the difficulties of not being able to present the items in full, we have shown that in the scenario, together with the multiple choice possibilities that follow, there is a rich set of mathematical ideas or concepts connected to the item. Our mathematical analysis of the LMT items shows their mathematical potential, precisely in the combination of a mathematics problem and a varied set of possible solutions, not all of which are correct. The item provides the possibility of exploring a connected sets of ideas with teachers, and so an important element of their SMK. Working productively with quadratic equations, for example, rests on

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<sup>8</sup> We can posit here (though this is beyond the scope of this paper) that the kind of analysis we have done enables us to develop a two-dimensional continuum across which we could provisionally place teachers' responses in this interview, and that can be contrasted with their responses on these items at a later point, and after participation in the project. We are still in the process of developing this continuum and clear descriptions of points related to PCK and SMK as these emerged in the interview setting. The placing of teachers would, of course, need to include awareness of where we did not have data in these initial interviews, and disclaimers that these are not 'measures' of teachers' knowledge, but qualitative indicators of knowledge in use in an interview setting. This assessment will combine with other teacher data in the project.

key ideas like: divisibility by a real variable and so too by zero; square roots; as well as a procedure that can be used to solve these equations.

Secondly, and linked to the first point, is that structuring the interview so that each of the four multiple choice options is discussed not only opens the space for teachers to talk, but it provokes directed talk and related reasoning. Looking across the responses of the teachers, both in the Appendix and Table 1, and the detailed data for Scenario 1, we see that the LMT items, as we used them, provide the possibility for teachers to talk about the range of mathematical ideas embedded in the item. This possibility was not evenly realised, in many cases because the interviewing was uneven. Not all interviewers deliberately asked teachers to focus on each of the four options in each of the scenarios. What we have learned from our analysis is that optimising the potential of these items in an interview setting requires a semi-structured interview that is explicit in what the interviewer needs to ask and then probe. It is possible, with greater structure and consistency in the interview, to create the conditions that all interviewees engage with and have to justify their thinking in relation to a connected set of ideas. Furthermore, our analysis of Scenario 2 suggests that it might be necessary to further structure the interview when the item is more difficult and less familiar. A probe in the interview on Scenario 2, that steers teachers to generate the sequence for each problem before they relate this to the model, and consider its appropriacy, could provide for further SMK related talk. We thus contend that there is much to be gained from using carefully constructed multiple choice items in an interview setting.

What we point to above, is the importance of being explicit about what is being assessed in the scenario or item, and thus concur with Schoenfeld (2007) that the lack of specification of expected responses is a limitation to the measures work. In this work, we need to be clear, at least at the level of intention, at what it is we are attempting to access and assess. In the context of this paper, and Hill *et al.*'s call, development crossover requires such explication.

Thirdly, our use of the items has shown the complexity in marking out a hard boundary between PCK and SMK, and SCK (specialised content knowledge) in particular. LMT stands by its distinctive categories within MKT, while acknowledging that much work is needed for clearly delineating KCS (knowledge of content and students) in particular. What our interviews have shown is that in the interview setting, the situating of the subject matter

knowledge in teaching contexts, particularly if this was a familiar context, lead teachers to recruit pedagogical considerations into their rationales, and these shifted between KCS (thus knowing what to focus on because of errors learners will make e.g. they don't recognise the quadratic form), and/or KCT (how to teach so that learners discern the quadratic form and the related procedure). And this is not simply a function of the interviewer steering the talk in one or other direction. As noted above, for T19 and T10, their confidence with their pedagogical knowledge (T19 of her students thinking; and T10 with what must be emphasised in teaching), was highly visible in the interview. Only further probing could elicit whether their PCK emphasis in the interview was a mathematical avoidance strategy. The introduction of the item in the interview with T19, and its probing by the interviewer focused directly on interpretation of the options, without reference to the teachers' classroom. It would require skilled interviewing to focus attention on the mathematics in the item.

And again, Schoenfeld's comment has salience. LMT is not explicit in exactly what they are assessing and how the distractors provide for such assessment. Our analysis of Scenario 1, and then the teachers' responses to this suggests that it would be possible, perhaps even likely in a test setting that Bullet 4 can be selected by elimination, and not through the composite logical reasoning entailed in the statement. It is precisely these complexities of measurement that motivated our consideration in this paper, and of course, in our practice, of whether our use of the items is indeed 'fit' for our purposes. Our reading of SMK in use with respect to connections across a range of mathematical ideas and related reasoning across the interviews, convince us of the difficulty of being able to clearly mark out and measure each of SMK and PCK, and particularly aspects of SCK as defined by Ball, Thames and Phelps. That COACTIV and LMT use different interpretations and operationalisations of SMK and PCK in particular, require that as a field, we critically interrogate generalised conclusions about the inter-relation of these domains of professional knowledge.

Finally, our discursive orientation to knowledge cautions our descriptions by underscoring that what is produced as teachers' mathematical knowledge through our use of the items in an interview setting is an interaction between the item itself, the knowledge resources the teacher recruits into the interview, and the way in which the interaction in the interview unfolds. Assessments, wherever they are, and however they are used, are a discursive product, and not a transparent view into a teacher's 'mind'.

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Appendix

| No | School | Teachers | Gender | Interviewer | Variable |     |     | Real Number System |     |     | Quadratic recognition |     |     | Divisibility by zero |     |     | Square root |      |      | Demonstrating own methods to solve |      |      | Logic (double implication) |      |      |  |
|----|--------|----------|--------|-------------|----------|-----|-----|--------------------|-----|-----|-----------------------|-----|-----|----------------------|-----|-----|-------------|------|------|------------------------------------|------|------|----------------------------|------|------|--|
|    |        |          |        |             | MV+      | MV0 | MV- | MR+                | MR0 | MR- | MQ+                   | MQ0 | MQ- | M?+                  | M?0 | M?- | MSq+        | MSq0 | MSq- | MDm+                               | MDm0 | MDm- | MLg+                       | MLg0 | MLg- |  |
| 1  | I      | T1       | Male   |             | 1        |     |     | 1                  |     |     | 1                     |     |     | 1                    |     |     | 1           |      |      | 1                                  |      |      | 1                          |      |      |  |
| 2  | I      | T2       | Female |             |          | 1   |     |                    | 1   |     |                       | 0   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 3  | I      | T3       | Female |             |          |     | 1   |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 4  | II     | T4       | Male   |             |          |     | 1   |                    |     |     | 1                     |     |     | 1                    |     |     | 1           |      |      | 1                                  |      |      | 1                          |      |      |  |
| 5  | III    | T5       | Female |             | 1        |     |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 6  | III    | T6       | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 7  | IV     | T7       | Female |             |          |     | 1   |                    |     |     | 1                     |     |     | 1                    |     |     | 1           |      |      | 1                                  |      |      | 1                          |      |      |  |
| 8  | IV     | T8       | Female |             |          |     | 1   |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 9  | V      | T9       | Female |             | 1        |     |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 10 | V      | T10      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 11 | VI     | T11      | Female |             | 1        |     |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 12 | VI     | T12      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 13 | VII    | T13      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 14 | VII    | T14      | Male   |             |          |     | 1   |                    |     |     | 1                     |     |     | 1                    |     |     | 1           |      |      | 1                                  |      |      | 1                          |      |      |  |
| 15 | VIII   | T15      | Female |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 16 | VIII   | T16      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 17 | VIII   | T17      | Female |             |          |     | 1   |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 18 | IX     | T18      | Female |             | 1        |     |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 19 | IX     | T19      | Female |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 20 | IX     | T20      | Female |             | 1        |     |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 21 | IX     | T21      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 22 | X      | T22      | Female |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 23 | X      | T23      | Female |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |
| 24 | X      | T24      | Male   |             |          | 1   |     |                    | 1   |     |                       | 1   |     |                      | 1   |     |             | 1    |      |                                    | 1    |      |                            | 1    |      |  |

review analysis of 24 teachers



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# The conceptualisation and measurement of pedagogical content knowledge and content knowledge in the COACTIV study and their impact on student learning

Stefan Krauss and Werner Blum

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## Abstract

An ongoing question is the extent to which teachers' professional knowledge has an impact on their teaching and, in particular, on their students' achievement. The COACTIV<sup>1</sup> study surveyed and tested the mathematics teachers of the classes sampled for PISA 2003/04 in Germany. The study's key components were newly developed tests of teachers' pedagogical content knowledge and content knowledge. This article gives a report of the conceptualisation and operationalisation of both domains of knowledge and describes the construction of the COACTIV tests. Findings from the tests show that there are differences with respect to both knowledge domains regarding teachers' school types, but that pedagogical content knowledge and content knowledge astoundingly both do not depend on teaching experience. Furthermore we show that the two domains of knowledge correlate positively with constructivist teachers' subjective beliefs, on the one hand, and with some crucial aspects of their instruction, on the other hand. Finally, we show that pedagogical content knowledge – but not pure content knowledge *per se* – significantly contributes to students' learning gains.

## The COACTIV Study 2003/2004

Although the essential influence of teachers on students' learning is obvious, empirical studies which assess aspects of the teachers' professional knowledge systematically, and link them with the students' achievement, are

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<sup>1</sup> COACTIV was a collaborative project, running 2002–2008, based at the MPI Berlin (Max-Planck Institute for Human Development; project director: Jürgen Baumert, project staff: Stefan Krauss, Mareike Kunter *et al.*), with the Universities of Kassel (director: Werner Blum) and Oldenburg (director: Michael Neubrand) as partner institutions. The COACTIV study was funded by the German Research Foundation (DFG) as a component of its BIQUA priority program on the quality of schools; see Kunter, Baumert, Blum, Klusmann, Krauss and Neubrand (forthcoming) for more details.

very rare. The main goal of the German COACTIV study (**C**ognitive **A**ctivation in the Classroom: Professional Competence of Teachers, Cognitively Activating Instruction, and Development of Students' Mathematical Literacy) was the investigation and testing of mathematics teachers of German PISA classes. The international PISA<sup>2</sup> study 2003, whose main focus lay in the subject of mathematics, has been extended in Germany both to a study based on whole classes (220 altogether) and to a longitudinal study, which means that the students of the grade 9 classes which were tested in PISA 2003 were examined again in grade 10 in the following year. Following this pattern, the COACTIV study investigated the mathematics teachers who taught these PISA classes in grade 9 and grade 10 at both PISA study dates (April 2003 and April 2004; therefore "COACTIV 03/04").

The COACTIV study 03/04, together with PISA, offered a unique opportunity to collect a broad range of data about both the students and their teachers, and to analyse them mutually. Due to the data of the COACTIV study it is not only possible to get an idea of the competencies and experiences of German secondary mathematics teachers, but it is possible to identify characteristics of a teacher empirically as well, which are relevant for the learning progress of students (or for different target criteria of mathematics lessons). In the context of the COACTIV study, numerous instruments for the investigation of mathematics teachers were newly developed or adapted (they include the measurement of professional knowledge, of motivational orientations, beliefs and values, aspects of work-life experiences etc.; a more detailed overview on that study is available in the book: "*Teachers' professional competence: Findings of the COACTIV research program*" (Kunter, Baumert, Blum, Klusmann, Krauss and Neubrand, forthcoming).

Figure 1 illustrates various aspects in which COACTIV collected data together with PISA. Together with the instruments which were used in PISA to examine the students, the teachers were presented with both questionnaires (regarding biography, interests, beliefs and more) and tests (e.g., regarding professional knowledge) in COACTIV. But what should a 'test' for teachers look like? With which knowledge should mathematics teachers be equipped?

From the point of view of mathematics, the pedagogical content knowledge (PCK) and the content knowledge (CK) are of special interest as central parts

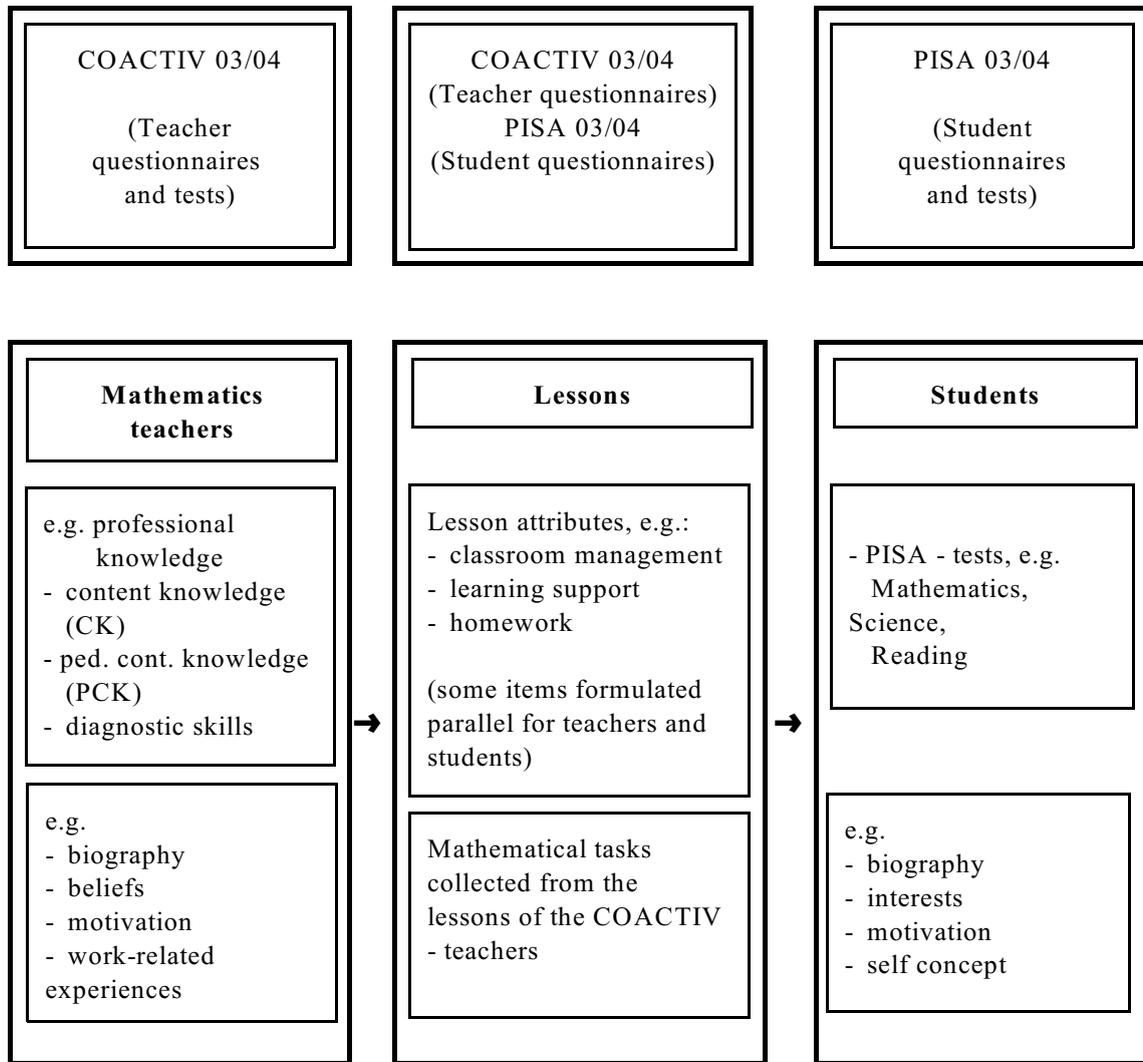
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<sup>2</sup> The OECD Programme for International Student Assessment, see <http://www.oecd.org/pisa/>

of the professional knowledge base (see Figure 1, left column). In the context of the COACTIV study, tests for mathematics teachers were developed for both knowledge categories which form the core of this study and which will be presented in the present paper in more detail.

In the above-mentioned book (Kunter *et al.*, forthcoming) the interested reader can learn more about results of other aspects which have been examined in the COACTIV study, for example, about the teachers' experience of stress and 'burn out', about enthusiasm or about beliefs (see also left column in Figure 1), about aspects of mathematics lessons in PISA classes from the point of view of teachers and students, and about the mathematics tasks used by teachers (middle column in Figure 1). Interesting results about students (right column in Figure 1) can be taken from the respective PISA book (OECD, 2004).

**Figure 1: Conceptual connection of the COACTIV 03/04 study and the PISA 03/04 study and sample aspects of three columns examined: mathematics teachers, mathematics lessons and students**



## Professional knowledge of mathematics teachers

At the outset of studies about teachers in the first half of the last century, the notion of *personality* was in the foreground. From the 1950s onwards, it was the teachers' *behaviour* in particular which was the object of research. Today it is the general opinion that above all the *professional knowledge* of teachers plays a crucial role in the regulation of behaviour and therefore in the control of the teaching and learning processes (a famous quote from Elbaz, 1983, says: "The single factor which seems to have the greatest power to carry forward our understanding of the teachers' role is the phenomenon teachers' knowledge."). Note that 'knowledge' here cannot be identified with declarative knowledge, in fact it must, in large parts, be regarded as procedural knowledge as well (routines, skills, abilities, competence) (cf. Weinert, Schrader and Helmke, 1990).

However, with which knowledge should teachers be equipped? The theoretical structuring of the teachers' knowledge into distinguishable categories is traced via so-called taxonomical approaches. One of the most influential knowledge taxonomies for teachers is the one of Lee Shulman (1986). Shulman introduced, among other categories, the domains of *pedagogical knowledge*, *content knowledge* and *pedagogical content knowledge*. These three categories form, seen from a contemporary point of view, the generally accepted core categories of the teachers' professional knowledge (e.g., Kunter *et al.*, forthcoming).

Considering the teachers' professional knowledge, lots of questions have remained unanswered over a long time: When is this knowledge acquired? How can it be measured? How does this knowledge influence lesson planning and the learning progress of students? An empirically valid answer to these questions requires that the relevant knowledge categories are made measurable. COACTIV sought to fill the gap in research concerning the two special knowledge categories, the pedagogical content knowledge and the content knowledge of mathematics teachers. Pedagogical knowledge (including the knowledge for the optimisation of the teaching-learning situation in general, e.g., classroom management, lesson structure, time management, discipline and the like), which should be essentially the same for teachers of different subjects, is not addressed in the present paper. A corresponding test construction for pedagogical knowledge has been

developed (see Kunter *et al.*, forthcoming) in the context of the follow-up study COACTIV-R (in which trainee teachers have been assessed).

How can pedagogical content knowledge and content knowledge of mathematics teachers be conceptualised? In the following, we introduce Shulman's (1986) characterisation, which forms the base of the test construction in the COACTIV-study.

### Pedagogical content knowledge (PCK) of mathematics teachers

In simple terms, Shulman (1986) defines pedagogical content knowledge as knowledge about "making content accessible". The core meaning of pedagogical content knowledge can best be taken from Shulman's original quote:

Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganising the understanding of learners, because those learners are unlikely to appear before them as blank slates (p. 9–10).

Simply, Shulman describes two aspects of pedagogical content knowledge: on the one hand he emphasises the knowledge about explaining and representing ('the ways of representing and formulating the subject'), and on the other hand he underlines the importance of knowledge on subject-related student cognitions ('conceptions', 'preconceptions', 'misconceptions').

Attention should be paid to the fact that Shulman's description is true for every subject: teachers of all subjects should be able to represent content of their subject appropriately and should be conscious of typical misconceptions of students. It is well known that in mathematics lessons mathematical tasks play a decisive role (e.g. Christiansen and Walther, 1986; Neubrand, Jordan, Krauss, Blum and Löwen, forthcoming). Mathematical tasks offer efficient

learning opportunities, and the majority of time in mathematics lessons is spent solving mathematical tasks. A substantial knowledge base on the characteristics of tasks is therefore of particular importance in mathematics lessons. It has to be taken into account that by ‘knowledge about tasks’ we do not mean the ability to solve mathematical tasks, but we mean the pedagogical knowledge about the potential of tasks for the learning of students (i.e. the knowledge about what a task can contribute to the students’ successful knowledge construction).

Pedagogical content knowledge of the subject of mathematics was therefore conceptualised in COACTIV with three key components of knowledge:

- *knowledge about explaining and representing mathematical contents (‘E&R’)*
- *knowledge about mathematics related student cognitions (‘StCog’)*
- *knowledge about the potential of mathematical tasks (‘Task’)*

In Figure 2, test items can be found illustrating these three sub-facets of pedagogical content knowledge. In addition, a sample item of the test on content knowledge (see under **Content knowledge (CK) of mathematics teachers**) can be found there. The conceptualisation of the three sub-facets of pedagogical content knowledge was defined more precisely for the purpose of the operationalisation of test items in the following way:

*Explaining and representing (‘E&R’): Operationalisation based on lesson scenarios*

The student’s knowledge construction can quite often only be successful because of instructional guidance (e.g. Mayer, 2004). Mathematics teachers should be able to explain and represent mathematical issues in an appropriate way. When operationalising this aspect of pedagogical content knowledge, 11 situations in mathematical lessons were constructed in which direct support for local processes of understanding was necessary (see sample item ‘minus 1 times minus 1’ in Figure 2). As profound knowledge on mathematical representations means the availability of a broad range of explanations for mathematical problems, the knowledge on representations was thereby brought into focus.

*Mathematics related student cognitions ('StCog'): Operationalisation as knowledge about typical errors and difficulties of students*

In order to be able to teach adaptively, a teacher has to be equipped with knowledge about typical content-related student cognitions. Difficulties and errors, especially, reveal the implicit knowledge of the problem solver and therefore make cognitive processes noticeable (e.g. Matz, 1982). In order to utilise the students' errors and typical difficulties as a pedagogical opportunity, mathematics teachers must be able to identify, classify conceptually and analyse the students' errors. For operationalising pedagogical content knowledge about the students' cognitions, seven situations in mathematics lessons were constructed in which the students' errors and difficulties had to be identified and/or analysed (see sample task 'parallelogram' in Figure 2).

*The potential of mathematical tasks ('Task'): Operationalisation as knowledge about the multiple potential to solve mathematical tasks*

It has been pointed out repeatedly that mathematical understanding can be developed by comparing different solutions of mathematical tasks (e.g., Rittle-Johnson and Star, 2007). In order to make this issue accessible in lessons, mathematics teachers have to be able to recognise the potential of tasks for multiple solutions and they have to know what kind of structural differences are featured by these different solutions of a mathematical task. For operationalising pedagogical content knowledge about the potential of mathematical tasks, four mathematical tasks were chosen, each including an instruction for the teacher to explicate as many substantially different solutions as possible (see sample item 'square' in Figure 2).

The pedagogical content knowledge test in COACTIV therefore consists of three subtests, namely on knowledge on explaining and representing (11 items), knowledge on the students' errors and difficulties (seven items), and knowledge on multiple solutions of tasks (four items). Altogether, pedagogical content knowledge was assessed by 22 items.

Note that this conceptualisation (including the respective operationalisations) can easily be embedded into a simple model of mathematics lesson: mathematics lessons can – using the briefest phrasing – be taken as making mathematical contents accessible to students. Because of the sub-facets of the

COACTIV test for pedagogical content knowledge it is assured that each of the three pillars of mathematics lessons (contents, students, making accessible) is covered by one component of knowledge: the ‘content’ aspect is covered by knowledge of the potential of tasks (multiple solutions of tasks), the ‘students’ aspect is covered by knowledge of subject-related student cognitions (errors and difficulties of students), and the ‘making accessible’ aspect is covered by knowledge of ‘making contents comprehensible’ (explaining and representing). Of course, the present conceptualisation cannot cover the pedagogical content knowledge completely; it can rather be seen as an attempt to assess relevant facets of pedagogical content knowledge related to crucial aspects of teaching mathematics.

Other conceptualisations of PCK have been developed, in particular, by Grossman (1990), Ball, Hill and Bass (2005), Hill, Rowan and Ball (2005) or Tatto, Schille, Senk, Ingvarson, Peck and Rowley (2008). While Tatto *et al.* (2008) investigated, in the Teacher Education and Development Study – Mathematics (TEDS-M), teacher trainees and pre-service teachers, Ball *et al.* (2005) examined the Mathematical Knowledge Needed for Teaching (MKT) of practicing elementary teachers. It is interesting to note that all projects regard knowledge on explanations and knowledge on student errors as crucial aspects of teachers’ pedagogical content knowledge. All projects use the format of lesson scenarios, but whereas most of the TEDS-M items and of the items of the Michigan group (e.g., Ball *et al.*, 2005; Hill *et al.*, 2005) have multiple choice format, all PCK and CK items in the COACTIV study have an open-ended format, thus avoiding the problems typically associated with multiple choice items. The different approaches are compared in detail in Krauss, Baumert and Blum (2008) (also see the article by Adler and Patahuddin in the present volume, where the authors explicate the approach of Ball and colleagues).

### Content knowledge (CK) of mathematics teachers

Content knowledge is generally seen as a necessary but not sufficient requirement for pedagogical content knowledge (e.g. see Kunter *et al.*, forthcoming). Although nobody queries that profound content knowledge is an unalienable basic requirement for successful lessons, this category of knowledge is treated a lot less extensively in literature – in comparison to pedagogical content knowledge. Shulman’s conceptualisation of content knowledge says:

To think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter [. . .]. For Schwab (1978) the structures of a subject include both the substantive and syntactic structure. The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established. [. . .]. The teacher need not only to understand that something is so, the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied (p. 9).

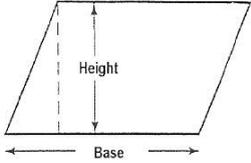
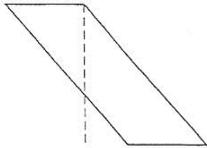
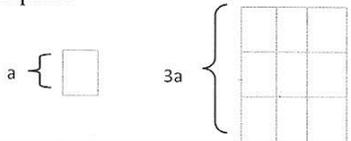
According to Shulman (1986), a teacher should be equipped – besides knowledge of mathematical facts – with the competence of argumentation and justification, e.g. for proofs or connections, within the discipline. However, Shulman’s description leaves the question open, with which level of content knowledge a teacher should be equipped in particular. Does he only mean the subject matter of the school curriculum, or is it crucial to have a broad basis of university-related knowledge available? The term ‘mathematical content knowledge’, in principle, can refer to the following different levels:

1. mathematical everyday knowledge
2. knowledge of the subject matter of the mathematical curriculum (contents which have to be learned by students)
3. advanced background knowledge of the subject matter of the mathematical curriculum
4. mathematical knowledge which is exclusively taught at university

‘Mathematical content knowledge’ was conceptualised in COACTIV on the third level, i.e. as advanced background knowledge about the subject matter of the mathematical school curriculum. In order to be able to cope with mathematically challenging situations in a lesson in a competent way, teachers are expected to conceive the subject matter that they teach on an appropriate level which is obviously above the common work-level in the lessons.

Altogether, 13 items with mathematics on an advanced school knowledge level were presented to teachers in the COACTIV test of content knowledge. For content knowledge, no sub-facets were postulated theoretically (but see, e.g. Blömeke, Lehmann, Seeber, Schwarz, Kaiser, Felbrich and Müller, 2008, for such sub-facets). A sample item of this COACTIV test can be found in Figure 2.

Figure 2: Sample items (and respective sample solutions) from the COACTIV tests of mathematics teachers' PCK and CK

| Category  | Sample items  | Sample solutions   |
|---|---|--|
| <p><i>Pedagogical content knowledge (PCK)</i></p> <p>“E&amp;R”</p>                            | <p>“Minus 1 times minus 1”</p> <p>A student says: I don't understand why</p> $(-1) \times (-1) = +1$ <p>Please outline as many different ways as possible of explaining this mathematical fact to your student.</p>   | <p>Although the principle of permanence does not prove that <math>(-1) \times (-1) = +1</math>, it could be used here to promote students' conceptual understanding and to establish mental connections between concepts:</p> $-1 \begin{cases} 2 \cdot (-1) = -2 \\ 1 \cdot (-1) = -1 \\ 0 \cdot (-1) = 0 \\ (-1) \cdot (-1) = 1 \end{cases} \begin{cases} \\ \\ \\ +1 \end{cases}$ |
| <p><i>Pedagogical content knowledge (PCK)</i></p> <p>“StCog”</p>                              | <p>“Parallelogram”</p> <p>The area of a parallelogram can be calculated by multiplying the length of its base by its height.</p>  <p>Please sketch an example of a parallelogram where students might not be able to apply this formula.</p> | <p>Students may have difficulties if the foot of the height is outside the parallelogram:</p>   |
| <p><i>Pedagogical content knowledge (PCK)</i></p> <p>“Tasks”</p>                              | <p>“Square”</p> <p>How does the area of a square change when the side length is tripled? Show your reasoning.</p> <p>Please write down as many different ways as possible of solving this problem (with explanations).</p>  | <p><u>Algebraic:</u></p> <p>Area of the original square: <math>a^2</math><br/>         Area of the “new” square: <math>(3a)^2 = 9a^2</math>, that is 9 times larger.</p> <p><u>Geometric:</u></p> <p>Nine times the size of the original square</p>    |
| <p><i>Content knowledge (CK)</i> (This item was taken from the pilot phase of the study.)</p> | <p>“Prime number”</p> <p>Is <math>2^{1024} - 1</math> a prime number?</p>   | <p>No, because:</p> $a^2 - b^2 = (a - b)(a + b).$ <p>Therefore, <math>2^{1024} - 1</math> can be broken down into <math>(2^{512} - 1)(2^{512} + 1)</math></p>  |

## Administration of the tests

Altogether, 198 mathematics teachers were examined with both the pedagogical content knowledge test and the content knowledge test. As the tests were administered at the second measurement date of COACTIV in 2004, the participating teachers were recruited from the mathematics teachers of the grade 10 classes which were examined for the German longitudinal PISA 03/04 component. Thus, this sample can be regarded as representative. In Germany, all candidates entering a teacher training program must have graduated from the highest track in the school system, the so-called 'Gymnasium' (Gy), and received the so-called 'Abitur' qualification (corresponding to the Grade Point Average in the USA). At university, those aspiring to teach at the secondary level must choose between separate degree programs qualifying them to teach either at Gy or in the other secondary tracks (e.g., 'Realschule' or 'Sekundarschule'). Gy and non-Gymnasium (NGy) teacher education students are usually strictly separated during their university training. One of the main differences in their degree programs is the subject matter covered: Students trained to teach at Gy cover an in-depth curriculum almost comparable to that of a master's degree in mathematics. Relative to their colleagues who receive less subject-matter training (and usually spend less time at university), Gy teachers may therefore be considered mathematical experts. NGy teachers, in contrast, study less subject matter but they are trained with more pedagogical content in university. 85 of the 198 teachers who were working on the tests were teaching at Gy (55% of them were male), and 113 of those were teaching at NGy (43% of these were male). The average age of the participating teachers was 47.2 years (with a standard deviation of 8.4); the teachers received an expense allowance of 60 Euro for their participation. The tests were administered in a single session with attendance of a trained test guide, normally in the afternoons of the PISA test day in a separate room of the school. For completing the tests, there was no time limit. On average teachers spent two hours to complete both tests (about 65 min for dealing with the 22 items of the pedagogical content knowledge test and about 55 min for dealing with the 13 items of the content knowledge test). The use of hand-held calculators was not allowed for the completion of the tests.

All of the 35 items were open questions. An instruction for coding was developed, and eight of the best mathematics teacher students of the University of Kassel were instructed carefully in coding teachers' answers. Each answer of the teachers was then coded by two of those trained students

independently (whereby sufficient results of agreement have been achieved). The procedure of test construction (including the scoring scheme of the item ‘square’ as an example) and resulting psychometric test properties can be found in detail in Krauss, Blum, Brunner, Neubrand, Baumert, Kunter, Besser and Elsner (forthcoming).

## Results

Figure 3 gives an overview of the test results, divided into the two different German school types. Note that all results refer to our specific conceptualisations and operationalisations of mathematics teachers’ PCK and CK

**Figure 3: CK and PCK: means M (standard deviations SD) and empirical maxima by teacher group**

|                           | M (SD)<br>Gy (N=85) | M (SD)<br>NGy (N=113) | Effect size d<br>(Gy vs NGy) | Emp.<br>max.<br>Gy | Emp.<br>max.<br>NGy |
|---------------------------|---------------------|-----------------------|------------------------------|--------------------|---------------------|
| CK (13 items)             | 8.5 (2.3)           | 4.0 (2.8)             | 1.73                         | 1.3e+09            | 1.2e+08             |
| PCK (22 items)            | 22.6 (5.9)          | 18.0 (5.6)            | 0.80                         |                    |                     |
| <i>E&amp;R</i> (11 items) | 9.3 (3.4)           | 7.1 (3.2)             | 0.67                         |                    |                     |
| <i>StCog</i> (7 items)    | 5.8 (2.3)           | 4.3 (1.9)             | 0.71                         |                    |                     |
| <i>Tasks</i> (4 items)    | 7.5 (1.8)           | 6.6 (2.0)             | 0.47                         |                    |                     |

*Gy academic track teachers, NGy non-academic track teachers. According to Cohen (1992), d=0.20 is a small effect, d=0.50 a medium effect, and d=0.80 a large effect. All differences are significant at p<0.01*

One specific feature of the pedagogical content knowledge test has to be mentioned: while in every item of the test for content knowledge one point could be scored with each correct answer (the maximum score which could be achieved was therefore 13), in the pedagogical content knowledge test in 9 of the 22 items multiple answers were allowed (and even asked for, see Figure 2). Considering the sub-facet ‘E&R’, this was the case in 3 out of 11 items, considering ‘StCog’, in 2 out of 7 items and referring to ‘Task’, in all the 4

items. Therefore a theoretical maximum for pedagogical content knowledge does not exist, but an empirical one: 37 points, which were achieved by one teacher; that means she was able to solve all items correctly and therefore got one point for each, and that she provided, on average, 2–3 correct alternatives in the multiple tasks.

As expected, due to the quite intensive training in subject matter of Gy teachers, a major difference could be recognised between Gy teachers and NGy teachers in their content knowledge. Considering the pedagogical content knowledge, Gy teachers also achieved more points on average, especially due to their higher competence level considering students' errors and explaining and representing (see Figure 3). However, it should be noted that Brunner, Kunter, Krauss, Baumert, Blum and Neubrand *et al.* (2006) showed that, when CK is statistically controlled for (i.e., when only teachers with the same CK level are compared), the NGy teachers slightly outperform the GY teachers with respect to PCK. The following results are worth mentioning as well (see Krauss *et al.*, forthcoming, for more detailed results):

1. Basically, the development of professional knowledge in our conceptualisation seems to be completed at the end of teacher training: surprisingly no positive correlations between both knowledge categories, on the one hand, and teaching experience or age, on the other hand, could be found. Of course, this does not mean that there are no other aspects of teachers' competence that increase with age and experience, for instance certain routines of classroom management. It only means that this kind of knowledge assessed by the COACTIV tests is obviously acquired during the time of the teacher training already. Indeed, from an additional construct validation study an intense increase of both knowledge categories could be found from the beginning of university studies to the end of teacher education. However, in this construct validation study only cross-sectional data were gathered (the examined samples were, among others, students at the end of Gymnasium and teacher training students at the end of university; for details of this construct validation study see Krauss, Baumert and Blum, 2008).
2. A strong relation between pedagogical content knowledge and content knowledge exists. Such a correlation is in line with the theoretical assumption of Shulman (1987) and other authors that pedagogical content knowledge is a certain 'amalgam' of content knowledge and

pedagogical knowledge. The relationship between the two knowledge categories can be examined directly by calculating the correlation between PCK and CK, which in the COACTIV data was  $r = 0.60$  indicating that PCK seems actually to be built upon a reliable base of CK. Note that this connection was much stronger in the Gy group; indeed, modelling PCK and CK as latent constructs led to a latent correlation in the Gy group that was no longer statistically distinguishable from 1 (see Krauss, Brunner, Kunter, Baumert, Blum, Neubrand and Jordan, 2008). Why was this correlation less strong in the NGy group? Closer inspection of the teacher data revealed that about 15% of NGy teachers who performed very poorly on CK (e.g., scoring only 1–2 points) nevertheless showed above-average performance on PCK (note that all teachers worked on all items and that the content areas of the PCK items differs from the content areas of the CK items). In other words, although our data support the claim that PCK profits substantially from a solid base of CK, CK is only one possible route to PCK. The greater emphasis on didactics in the initial training provided for NGy teacher candidates in Germany seems to be another route.

3. Voss, Kunter and Baumert (forthcoming) theoretically and empirically analysed the structure of subjective beliefs of the COACTIV teachers by using a 2x2 table with the first dimension *nature of mathematical knowledge* opposed to *teaching and learning of mathematics* and the second dimension *transmissive vs. constructivist orientation toward learning*. They found that only constructivist orientation but not a transmissive orientation contributes positively to the quality of lessons. Krauss *et al.* (forthcoming) found, for example, that teachers with high PCK and CK scores tended to disagree with the view that mathematics is ‘just’ a toolbox of facts and rules that ‘simply’ have to be recalled and applied. Rather, these teachers tended to think of mathematics as a process permanently leading to new discoveries. At the same time, the knowledgeable teachers rejected a receptive view of learning (“mathematics can best be learned by careful listening”), but tended to think that mathematics should be learned by self-determined, independent activities that foster real insight (for the development of beliefs see Voss *et al.*, submitted; or Schmeisser *et al.*, forthcoming).
4. Because COACTIV was ‘docked’ onto the PISA study, it was possible to relate teachers’ PCK to their students’ mathematics achievement

gains over the year under investigation. Possibly the most important result of the COACTIV study is that pedagogical content knowledge – but not content knowledge itself – contributes to the quality of lessons and to the students' learning decisively. Very briefly, when their mathematics achievement in grade 9 was kept constant, students taught by teachers with higher PCK scores performed significantly better in mathematics in grade 10. By means of structural equation modelling, Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand and Tsai (2010) or Baumert and Kunter (forthcoming) could show that PCK can explain students' achievement gains in a substantial way. To be more precise, if the pedagogical content knowledge of the teachers differed by one standard deviation (which means about 6 points in the PCK test according to Figure 3), the mathematical achievement of their students differed by nearly two thirds of a standard deviation after one year of schooling (which is really a lot taking into account that the average learning progress in grade 10 in PISA was a third of a standard deviation). Because student learning can be considered the ultimate aim of teaching, the discriminant predictive validity of both knowledge constructs can be considered a main result of the COACTIV study, especially in the light of the high correlation between both knowledge constructs (for more details see Baumert *et al.*, 2010; or Baumert and Kunter, forthcoming).

## Summary and discussion

The construction of knowledge tests for teachers has been demanded emphatically (e.g., Lanahan, Scotchmer, McLaughlin, 2004). COACTIV focused on specifying pedagogical content knowledge and content knowledge for the subject of mathematics, constructing appropriate tests, and utilising them for a representative sample of German mathematics teachers of secondary schools.

Referring to the understanding of mathematics lessons as making mathematical contents accessible to the students, pedagogical content knowledge was conceptualised and operationalised in COACTIV as knowledge on explaining and representing ('making accessible' aspect), on errors and difficulties of students ('students' aspect) and on multiple solutions of mathematical tasks ('contents' aspect). Considering the content knowledge

test, it has to be emphasised that due to the chosen curriculum-focused conceptualisation (advanced background knowledge of school mathematics) no empirically verified statements about the importance of high-level content knowledge that in Germany is gained at university could be deduced. In order to be able to investigate the relevance of this high-level content knowledge for student learning, a new test construction would be necessary.

Essential results, considering pedagogical content knowledge and content knowledge of COACTIV teachers, are the following: teaching experience does not seem to make a relevant contribution to the development of the two knowledge domains, which suggests that pedagogical content knowledge and content knowledge (as conceptualised in COACTIV) obviously are primarily acquired during teacher training. In order to examine the exact time and process of the acquisition in both knowledge domains, more studies with the COACTIV tests are necessary (e.g. with teacher trainees or student teachers; see COACTIV-R). Gy teachers show higher scores on content knowledge. The fact that Gy teachers are equipped with significantly more PCK (even if the difference is less noticeable in this case than in the case of CK) can be taken as an indication of the importance of content knowledge for the development of pedagogical content knowledge. However, a small group of NGy teachers (about 15%) shows that it is possible to possess outstanding pedagogical content knowledge with less content knowledge.

A result of great significance is the fact that pedagogical content knowledge of a teacher – but not content knowledge *per se* – contributes substantially to the learning development of the students. Therefore it is worth investing in teacher training of mathematics teachers, especially with respect to pedagogical content knowledge, with a sound basis of content knowledge.

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# Mediating early number learning: specialising across teacher talk and tools?

Hamsa Venkat and Mike Askew

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## Abstract

In this paper we locate our work in the context of claims of poor performance amongst South African learners in primary mathematics, and gaps in the knowledge base of primary mathematics teachers. Our focus is on the analysis of three Grade 2 Numeracy teachers' actions with artifacts in the context of increasing resource provision in the South African national policy landscape. Using ideas of mediation drawn from the Vygotskian tradition, and developed by Michael Cole and Jim Wertsch, we identify actions with the artifacts that suggest shortcomings in teachers' understandings of the mathematical structures that underlie the design of the resources. Drawing on the work on modeling and tool use in the Dutch Realistic Mathematics Education tradition, we note disruptions in openings for pedagogy to provide 'models of' increasingly sophisticated strategies, which might provide children with 'models for' working more efficiently (and ultimately 'tools to' think with), disruptions arising from teachers presenting only concrete unit counting based models of early number calculations. Within a policy context where improving the resource situation is a priority, we argue for more attention to longitudinal support to teachers to develop understandings of number and its progression that allow them to see the significance of the mathematical structures that are figured into the design of the artifacts that are increasingly available for use.

## Introduction

Performance in mathematics at all levels of the schooling system in South Africa continues to be described in terms of a 'crisis' (Fleisch, 2008). The recently introduced Annual National Assessment (ANA) – standardised national tests taken by all primary grades in mathematics and language – continue to show very low levels of attainment in the Foundation (Grades 1–3) and Intermediate Phases (Grades 4–6), with, in 2011, national mean performances in the Grade 3 numeracy test standing at 28% and at 30% in the Grade 6 Mathematics test (Department for Basic Education, 2011b).

In this context, research and policy attention has shifted back to the primary years with a range of issues cited as factors contributing towards this crisis of

performance including lack of curriculum clarity and inadequate resources for teaching. Policy responses to this situation have resulted in a more prescriptive curriculum together with the provision of resources (artifacts such as abaci and hundred squares) within the Foundations for Learning (FFL) campaign, introduced in 2009. The policy of prescribing the content, sequencing and pacing – backed up by the increasing provision of standardised termly tests and resources – continues into the recent 2012 introduction of the mandatory Curriculum and Assessment Policy Statements (CAPS) (Department for Basic Education, 2011a).

In 2011 we began the 5-year research and development focused Wits Maths Connect–Primary Project, working with ten government primary schools in one district. Our observations during the first year suggested problems with take up of the resources provided. Crucially our data suggested that even in the context of tightly prescribed content coverage with provision of supporting artifacts, a lack of in-depth attention to teachers’ understandings of the mathematical structure of number embedded in their talk and in their use of resources to represent mathematical ideas often prevented coherent and progressively sophisticated opportunities to work with early number. In this paper we explore possible reasons for this particularly with respect to teachers’ pedagogic content knowledge.

## The policy context

In recent years the structure and policy presentation of mathematics curricula in the primary years has reverted to a format that incorporates prescription of content, sequencing, pacing and progression. In 2008, a Government Gazette signaled the introduction of the ‘Foundations for Learning’ (FFL) campaign (Department of Education, 2008a). This campaign incorporated the introduction of the ANA, alongside the prescription of a minimum of an hour of mathematics teaching each day – which was to include at least ten minutes focused on mental arithmetic skills. ‘Daily teacher activities for numeracy in Grades 1– 3’ were also detailed in this document, with the following activities specified for whole class work:

Count with whole class according to their level

- Count using a number square
- Count on the number line
- Count forwards and backwards
- Count forwards and backwards from a given number to a given number
- Count in multiples
- Odd and even numbers, etc. (p.17)

Coupled with these activities, a set of ‘Recommended resources for Numeracy in Grades 1–3’ was also specified (p.18). For display on walls and teacher whole class work, this list included resources like a number line, a large 100 square, and a large abacus. At learner level, the list included an individual 100 square, place value cards, counters, and an individual small abacus.

The FFL Assessment Framework (Department of Education, 2008b) also introduced the notion of termly curriculum ‘milestones’ – described thus:

The milestones (knowledge and skills) derived from the Learning Outcomes and Assessment Standards from the National Curriculum Statement for Languages and Mathematics (Grades 1–3) have been packaged into four terms for each grade to facilitate planning for teaching. These milestones explain the content embedded in the Learning Outcomes and Assessment Standards.

These milestones have been further written into manageable units to assist you to develop the required assessment tasks per term (p.iii).

Research findings from South Africa show however that provision of detailed curriculum statements and resources in and of themselves may not be a key policy lever for raising standards. The work of Ensor and colleagues (2009), for example, presents evidence of teaching that holds learners back through focusing on concrete counting based strategies rather than supporting the shift over time towards more abstract calculation based strategies. The analysis leading to this finding incorporated a focus on ‘specialisation strategies’ (following Dowling, 1998), which include specialisation of content and modes of representation. Specialisation of content in early number refers to tasks that shift over time from counting, to calculation by counting, to calculating without counting. Specialisation of modes of representation correspondingly relates to shifts from concrete representations towards more abstract symbolic representations of number. Ensor *et al.*’s findings indicated a high prevalence (89% of the total time across the Grades 1–3) of counting/ calculation by counting-oriented tasks. The authors noted that ‘concrete apparatus for

counting, and for calculating by-counting, is visible in all three grades, with sustained use through Grades 1, 2 and 3' (p.21).

These findings relating to teaching of early number are consistent with broader findings at the learner level which suggest the ongoing predominance of unit counting and repeated addition/subtraction strategies well into the late Intermediate Phase (Schollar, 2008).

Policy that provides artifacts with minimal induction into the form and function of such artifacts must either assume that teachers have the pedagogic content knowledge to use the resources in ways that move learners from unit counting, or assume that the specification for skills in the curriculum bridges the gap. However, a central thread of evidence in South Africa related to the crisis in performance points to weaknesses in primary teachers' mathematics content knowledge and pedagogic content knowledge (Carnoy, Chisholm, Chilisa, Addy, Arends, Baloyi *et al.*, 2011; Taylor, 2010), with this thread having significant international parallels (Ball, Hill, and Bass, 2005; Ma, 1999). Carnoy *et al.*'s (2011) study indicated low levels of content knowledge in relation to Grade 6 mathematics content, and significant positive associations between teachers' content knowledge, pedagogic content knowledge and the time they spent teaching mathematics. This study also noted specific weaknesses in what the authors referred to as 'mathematical knowledge in teaching', defined in the following terms:

the degree to which teacher can appropriately integrate the use of the instructional techniques with the mathematical concept being taught and its effectiveness on learner learning. This also includes the use of correct language to clearly convey mathematical ideas (p.102).

Within the broader field of study into mathematical knowledge for teaching, our focus here is on aspects of what Hill, Ball and Schilling (2008) have identified as aspects of 'specialised content knowledge', which includes the need to represent and explain mathematical ideas, and 'knowledge of content and students', which includes awareness of typical student development sequences.

## The research

In this paper we pick up on the issue of teachers' integration of 'instructional techniques' with learning outcomes. Our focus is on teachers' use of artifacts to mediate learners' developing number-sense. Through this focus we examine what this can tell us about teachers' content knowledge and pedagogic content knowledge (PCK).

Our data are drawn from three Grade 2 Numeracy lessons and, given the extensive focus on number in the Foundation Phase Numeracy curriculum, we emphasise meanings and strategies related to number sense. Analysis of teachers' practices indicated difficulties in using abaci and hundred squares – both resources advocated within the policy context – as effective mediating means. We argue that these difficulties are linked to limited pedagogic content knowledge regarding number structure and its presentation to learners. Our motivation for doing this work is twofold. First, we seek to understand misalignments between teacher talk, actions and artifacts to inform our work in the primary mathematics focused Wits Maths Connect–Primary project, and the work of others engaged in teacher development. Second, we aim to contribute to the policy process with evidence of some ways in which policy artifacts are being taken up in classrooms and what needs to be done to improve effectiveness within take-up.

We begin with a summary of the ways in which number sense is described in the international literature and some of the practices and artifacts advocated for teaching aimed at developing number sense. The analytical base that we draw from is based on the Vygotskian notion of artifact mediated action, and follow-up work that distinguishes between artifacts taken up in the ways that build on the meanings connected to the cultures from which they are drawn (ideality) and artifacts taken up in ways that relate to other aspects of their materiality. We do this by linking the work of post-Vygotskian theorists such as Wertsch and Cole with the theory linking models and tools arising from the Freudenthal institute.

Following an outline of the data sources, we present three episodes of teaching involving the use of policy-advocated artifacts and provide an analysis of these episodes based on concepts related to the theory of mediated action. In each case we argue that the teachers' work with the artifacts

proceeds in ways that are unlikely to support the development of learners' number sense. We conclude with a discussion of the ways in which our evidence suggests the need to move beyond both mere provision of classroom artifacts and claims of lack of content/pedagogical content knowledge, and instead to move towards longitudinal teacher support based on the use of artifacts in ways that build on their form and relate to the meanings of number, operations and problem-solving.

## Number sense and teaching for number sense

McIntosh, Reys and Reys' (1992) widely cited paper describes number sense in terms of three key areas: knowledge and facility with numbers, knowledge and facility with operations, and applying this knowledge and facility with number and operations to a range of computational settings (p.4). Application skills based in strong number sense would demonstrate use of operations in ways that are well connected both to the numbers being operated on, and to the problem context. A recurring feature of the application category is the requirement for 'flexibility' and 'efficiency' of strategy – a point echoed by Kilpatrick, Swafford and Findell (2001) in their description of strategies backed by mathematical proficiency as being both 'effective' and 'efficient'. Flexibility depends on being able to adapt selected strategies to a range of problem settings. Central to the building of efficiency in early number learning is the adoption of 'non unit counting' strategies, which in turn, involve the reification of at least some counting processes into numerical objects (Sfard, 2008), or in the words of Gray (2008): "a shift in attention from the objects of the real world to objects of the arithmetical world – numbers and their symbols" (p.82). Such reification allows initially for 'count on' based strategies, and subsequently for the creation and use of strategies based on calculating with grouped numbers – using '5' and '10' in particular within early number learning as benchmarks for calculating 'through'.

Writing focused on progression within number sense has noted the importance of representations in supporting this reification of counting processes into numerical objects. Askew (1998) draws a distinction between 'structured' and 'unstructured' materials – with the former viewed as resources that are 'usually designed to embody a particular mathematical idea' (p.13). In contrast, 'unstructured' resources tend to be everyday materials or objects used for counting or measuring. Structured resources include artifacts

such as dice that support ‘subitizing’ (the immediate perception of small quantities without unit counting) and artifacts that provide experience of a range of representations showing ‘pair-wise’, ‘five-wise’ and groups of ten based number representations (Bobis, 1996; Gravemeijer, Cobb, Bowers, and Whitenack, 2000) that can support later understanding of operations within the decimal number system. In this view, artifacts such as abaci and 100 squares that provide visual representations of number in the 1–100 range as constitute-able through a quantity-based decomposition into tens and units – e.g. seeing 54 as constituted by 50 and 4, with the 50 further broken down into five 10s (Thompson, 1999) – would be structured resources, whereas cubes and counters would tend to fall into the unstructured resource category.

Given the widespread acceptance of the importance of actions on objects forming the basis for well-connected understandings of number ideas, artifacts that can be operated on, and that lead into and link with representations of these ideas, are viewed as critical to early mathematical learning. Thus there is wide agreement that resource provision is an important aspect of supporting the development of numeracy. But whilst representations in early number learning must include actions on objects, early number sense has been described as needing to go beyond this to develop well-connected networks between the language of a problem, the actions on objects that physically ‘replay’ or model the problem, and the diagrams and representations that provide a record of these actions or models, leading eventually to symbolic representations (Haylock and Cockburn, 2008). In turn, and related to progression in terms of representations, research points to limited number sense being related to representations that are closer to concrete actions and good number sense being related to more compressed symbolic representations (Askew and Brown, 2003). Closely related to this progression of representational forms, Steffe, Von Glasersfeld, Richards, and Cobb (1983) note that progression in counting strategies can be related to being able to work across a range of count-able units with perceived items at the lowest level through to abstract number units at the highest level of competency.

Across all these discussions, a common theme is the idea of teachers modelling coherently connected and increasingly sophisticated and efficient strategies for solving number problems, using suitable artifacts and meditating the key structures within these artifacts to represent their problem-solving processes. This teacher modelling provides the basis from which learners can appropriate and use similar actions for themselves, actions that

latterly become internalised psychological constructs for learners to work with. In the language of the research coming out of the Freudenthal institute (Gravemeijer and Stephan, 2002), the teacher initially provides a *model of* how to act on and talk about artifacts. For example, a child adding eight and four might count out eight and then count on four. The teacher provides a model of how this can be done on the abacus by using the ten structure to slide across eight beads without counting them in ones and also modelling how the four could be added as two followed by two. There is no suggestion in this *model of* that what the teacher shows is an accurate mirroring of what the child does: the child's actions provide the impetus for the teacher's actions, but the teacher shows how the artifact can be acted on in ways that mirror the child's actions and, where possible, drawing attention to how the artifact may be acted upon more efficiently. Thus increasingly sophisticated strategies are related to gradually more compressed representations, which in turn, need to be based on more compressed actions on mediating artifacts.

As the children watch the teacher frequently act and talk in such ways, they begin to appropriate the actions and language for themselves, acting on the artifacts in similar ways to the teachers' modelling: the artifact becomes a *model for* the learner to operate on. Ultimately, if the artifact is well designed (for supporting reification and mental 'actions'), this experience of acting on material artifacts supports the development of *tools for* thinking whereby the learner has internalised the language and actions to the point of being able to operate in such ways without the physical presence of the artifacts. (The theory does not suggest that this interiorisation of *tools for* is a result of a simple internalisation of the physical action, but studies do suggest that models such as the number line do fit with how the brain may process quantities additively (Dehaene, 1999).)

## Analytical concepts

The central focus of this paper is on three excerpts of Grade 2 Numeracy teaching in which artifacts were deployed to support problem-solving on a given task (the problems being numerical calculations the answers to which the learners do not have rapid recall). Wertsch's (1991) concept of mediated action, drawn from his studies of Vygotsky's notion of mediation (Vygotsky, 1981), is used to analyse teacher actions effected using selected tools to solve a problem they have set. For Wertsch (1995), mediated action rests on

‘individual-operating-with-mediational-means’ (p.64) as the central unit of analysis. In this linked agent-artifact formulation, the possibilities for transformation of action are present but Wertsch emphasises the need for agent take up of the artifact:

this is not to say that the mediational means somehow act alone. An individual using the new mediational means had to change as well, since it obviously called for new techniques and skills (p.67).

In the first instance here, the teacher is the individual using the ‘new mediational means’ and, as argued above, the use has to provide an appropriate *model* of the use of the artifact. There is little point in teachers acting on, say an abacus, in ways that do not make use of the form and function of the artifact, or in ways that could have been done with an unstructured collection of discrete objects. This combined focus on agent-artifact is central to our focus on ruptures in the process of take up (in effective ways) of the resources being inserted via the policy landscape, ruptures that appear to relate to a lack of use of the mathematical structure that underlies the design of the selected resource, which in turn is linked to teachers’ PCK.

Cole’s (1996) distinction between ‘artifacts’ and ‘tools’ is useful here, and he draws on the distinction made by Bakhurst (1991) between the ‘materiality’ and ‘ideality’ of artifacts to make his point. Bakhurst notes that:

as an embodiment of purpose and incorporated into life activity in a certain way – being manufactured for a reason and put into use – the natural object acquires a significance. This significance is the ‘ideal form’ of the object, a form that includes not a single atom of the tangible physical substance that possesses it (p.182).

The ‘physical substance’ constitutes the ‘materiality’ of the artifact. For Cole, an artifact is transformed into a tool when the nature of its use corresponds with the purposes recognised by the culture – i.e. when its use embodies ideality rather than materiality. Tool use, rather than simply artifact use, is associated with the transformation of mental functioning, following Vygotsky’s (1981) formulation:

The inclusion of a tool in the process of behavior (a) introduces several new functions connected with the use of the given tool and with its control; (b) abolishes and makes unnecessary several natural processes, whose work is accomplished by the tool; and alters the course and individual features of all the mental processes that enter into the composition of the instrumental act, replacing some functions with others (i.e. it re-creates and

reorganizes the whole structure of behavior just as a technical tool re-creates the whole structure of labor operations) (p.139–140).

The Dutch research inserts a stage into this movement of artifact to tool, from materiality to ideality: the learner's use of an artifact as a model for carrying out an operation. At this stage the boundaries between artifact and tool are blurred: two learners could be acting in similar ways, but one might simply be mimicking what the teacher has done, while the other could have come to appreciate the ideality of the artifact and be closer to appropriating it as a tool for thinking. The analysis of this movement is beyond the scope of this paper, but we draw attention to it to highlight the importance of teachers appreciating the ideality of artifacts and providing appropriate models of how to act on them.

Our interest in doing the analysis, within the context of the teacher development work in this project, is to understand the kinds of tasks, artifacts and experiences over time that we might need to provide teachers with access to in order to build settings within which teachers' understandings are attuned to the mathematical structures that the artifacts are intended to model that are starting to become more available through the policy landscape.

## Data sources

The data presented in this paper is drawn from our 'baseline' observations of Grade 2 Numeracy teaching at the start of 2011 as we launched the Wits Maths Connect–Primary (WMC-P) project. We observed, videotaped and transcribed single lessons taught by 33 of the 41 Grade 2 teachers across the ten schools in our sample – with a view to more broadly gaining insights about the nature of teaching and learning across our schools, and the contexts of teachers' work. In this paper three episodes from these observations are selected because they are typical of a number of lessons observed that showed evidence of artifacts being used in ways that seemed to bypass the mathematical structures built into their design. In all cases we have other instances in our data of the same kind of actions being enacted by other teachers in the Grade 2 sample, suggesting the wider presence of the phenomena of rupture between task, artifact and talk that we seek to describe and analyse here.

## The excerpts

### Excerpt 1

Ms. C works in a township/informal settlement school where mathematics is taught in Home Language. Ms. C uses a mix of Tsonga and English in her teaching, and often stresses vocabulary across the languages to support children's learning of mathematical vocabulary in both. In the focal lesson, Ms. C announced: 'Today we are going to do subtraction'. The vocabulary of subtraction actions in English and Tsonga is introduced in terms of 'minus'/'asoosa'/'minus, we take away something and throw it away'. A large abacus sat on the teacher's desk and learners each had a small abacus in front of them.

The series of examples listed below was worked through, some as whole class demonstration (WC) with the teacher modeling actions on her large abacus, and some set as seatwork tasks for individuals to complete (SW):

$$5 - 3 \text{ (WC)} \quad 4 - 0 \text{ (WC)} \quad 6 - 2 \text{ (SW)} \quad 8 - 5 \text{ (SW)} \quad 10 - 5 \text{ (SW)}$$

Across each of the whole class demonstrations, unit counting of beads on the abacus was used to count out the first number, and then to 'take away' the second number, and then to count out, again in ones, what was left. In the seatwork tasks, some learners were seen also to be unit counting in this way to solve the problems, whilst others appeared to be adding the two numerals rather than subtracting. Ms. C went round checking learners' abacus representations of each calculation and the actions they enacted to produce answers. She noted the addition representations and emphasised: 'We are not adding'.

Following these examples, Ms. C announced in English, and then in Tsonga:

*Now we are going to minus bigger numbers of two digits minus-ing only one digit. Nothing is changing when we minus the bigger numbers. You are still going to use your abacus and count what you will be given, right.*

She wrote 12-4 on the board, and then said:

*Let me show you how we count two numbers that we must minus with only one number. Let's count 12 balls.*

Twelve beads were counted out in ones, as before, with the count now using two rows of the abacus; ten on the top row, two on the one below. She asked at this point:

*How many lines did I use?*

The class responded: 'Two', to which Ms. C replied:

*Two lines because it's twelve. It has to jump to another row right?*

The teacher then proceeded to take away four through unit counting, sliding back, the two beads (one at a time) on the second row and then two (again singly) on the top line. The remaining beads were then counted out one by one to get eight.

### Excerpt 1 analysis

The abacus is the key mediating tool in use in this excerpt. The structure of the abacus – with ten rows each carrying ten beads – provides a physical form of the '10-based' structure of numbers to 100 in the decimal system. However, almost no recourse is made to this structure within the strategies presented by Ms. C. Further, as noted by Ensor *et al.* in their study (2009), all the calculations presented are solved through unit counting processes involving 'count all' strategies.

Two key questions emerge from this initial commentary. The first, related to the lack of take up of the '10-ness' structure of the artifact, is whether the abacus was an appropriate tool in relation to the tasks set? On the one hand the calculations set could be answered with an unstructured collection of discrete objects – which is essentially the way in which the abacus is being used here (the fact that they are beads on a rod simply making it easy to keep track of the unit counting). This would be classified as take up of the materiality, rather than the ideality, in Bakhurst's (1991) terms of the mediating artifact (and consequently, limiting the potential for this laying the basis of a psychological tool). Clearly, for some learners in the class who did not yet appear to have made sense of subtraction as an operation artifacts that

allow for the building of a connection between the ‘taking away’ action being presented here and the subtraction problem in word and symbolic form, would seem to be the priority.

The second question relates to whether the structure of 10 was worth pursuing for this set of calculations, most of which involved subtracting single digit numbers from (at most) two digit numbers less than 20. The unit count strategy effectively ‘produces’ the answers to all of these questions. Ms. C’s consistent use of unit counting suggests that ‘getting the answer’ was the object of her mediated action, but this leaves aside the notions of flexibility and efficiency of strategy identified as important in the number sense literature. Greater efficiency is certainly possible with a grouped count strategy, and the structure of the abacus would support the modelling of 12 in the last calculation (12–4) as a composite number made up of 10 and 2, each of which could be moved across on the abacus as grouped objects with a single swift action rather than in a one-by-one fashion. Ms. C drew children’s attention to the fact that 12 required beads from 2 rows of the abacus, but made no explicit reference to the breakdown into a ten and units. Her expression of this idea therefore remained at an everyday, rather than at a more specialised level based on the decimal structure of number. Further, her actions created the ten and unit values through counting processes rather than as conceptual objects in Gray’s (2008) terms. The teacher’s actions were limited in terms of being a ‘model of’ that might provide the basis for learners’ later use of the abacus as a ‘model for’ calculating using the base-ten structure.

For efficiency to be pursued, it would appear that teacher and learners require prior experiences with moving interim (1–10) numbers of beads with a single shift, in ways that would support reifications of smaller numbers that could lead into discussion of the 10-ness structure of the abacus, and the ability to very quickly represent 12 as a quantity on an abacus using this structure. In this more nuanced artifact mediated action we see possibilities associated with working with learners at various stages of the counting, calculating by counting, and calculating hierarchy – and actions that learners could appropriate with different degrees of understanding thus encouraging the artifact/tool and material/ideal shifts noted by Cole (1996). Thus whilst Ensor *et al.* point out shifts in specialisation of representation from concrete to symbolic representations, we suggest that there are gradations *within* these representational categories as well that can relate to building calculating that is likely to involve some reified number facts and some counting – a feature

noted as prevalent in children's work with number across a range of attainment (Thompson, 1995).

### Excerpt 2

In Ms. K's classroom, a mix of Zulu interspersed with English was used within a lesson focused on addition. The following episode occurred early in the lesson. Ms. K had distributed a worksheet with the following table drawn at the top, and two further similar tables (with different addends) below it:

|   |   |    |   |    |    |    |
|---|---|----|---|----|----|----|
| + | 8 | 16 | 4 | 20 | 12 | 24 |
| 4 |   |    |   |    |    |    |

Each learner had an individual abacus in front of him or her, and the teacher had a large abacus at the front. The following explanation unfolded for how to work out answers using the abacus.

T: Let us do Number 1. We read it together [pointing on the board].

T & class: 4 plus 8. Ha! [Teacher picked up the large abacus that had eight beads showing on the top row from a previous unit count out activity. Teacher held the abacus up and moved back the eight beads before counting out four beads through unit counting with the class.]

T & class: 1, 2, 3, 4. Is that correct?

Class: Yes.

T: Plus 8. [With the class chanting, eight beads were counted out one by one on the second row of the abacus.] 1, 2, 3, 4, 5, 6, 7, 8. Now before you do it, let's work it out. [Eight beads in the second row were counted again with whole class chant in unit count followed by '9, 10, 11, 12' as the first row beads were added in.] Twelve! Which means the answer is – ?

Class: Twelve.

T: The answer is twelve. [Teacher wrote 12 into the box beneath 8 on the worksheet copy stuck on the board.]  
Subsequently,  $4 + 16$  was also worked out using a unit count of beads across three rows of the abacus – four on the top row, ten on the second and six on the third, as were the remaining sums in the table.

### Analysis of Excerpt 2

This excerpt is similar in many ways to excerpt 1, and also based on the use of the abacus artifact. We see once again that mediation relied on unit count based strategies to represent both addends, and counting all to produce the answer. However, the number range was higher, and combined with addition as the operation in focus, resulted in some answers going above 20.

A difference of interest relating to artifact use in this episode is the way in which the addition was represented on the abacus. For example  $4 + 8$  was represented by four beads pulled across one by one on the top row, followed by eight beads pulled across on the second row. In Ms. C's excerpt, subtraction proceeded by taking (in  $12 - 4$ ) the two on the second row away followed by two more on the first row, which is close to modeling  $12 - 4$  as  $12 - 2 - 2$ . In contrast Ms. K's modeling, where the two numbers were pulled from two different rows, disrupts the possibility of bridging through 10 at all. Therefore, although the numbers being operated on are larger, in one sense, the mediation in this excerpt is more rooted in the realm of concrete counting than in Excerpt 1.

### Excerpt 3

Ms. S worked in a township/informal settlement school and taught a mixed Grade 2/3 class. Within the focal lesson, opening activities were based on whole-class chanting of days of the week, and 10s to a 100. The class was then asked to use fingers and/or the individual 100 squares they had on their desks to count in 1s, 2s and 5s to 100 – something that not all learners appeared able to do.

At this stage, pointing to the large 100 square stuck on the blackboard, Ms. S announced:

*I want to see if you know the number that you count, I'm going to hide a number and I want you to show or tell me [. . .] On our chart I will hide one number and you tell me which I hide.*

She stuck a small blank square over the number 15 on her 100 chart and asked the class which number was covered. She prefaced learner answers with: 'Did you see which number I hid?' 'Fifteen' was audibly called out, but the first two children the teacher asked said the answer was 'Fifty'. The third child offered 'Fifteen', and the class, in chorus, agreed that this was correct. The blank square was then stuck over the number 44, and the same question posed: 'Did you see which number I hid?' The following interaction then ensued:

Lr1: 34

T: Is it 34?

Class: No

T: [...] Is she correct [pointing at another learner]?

Lr2: 24

T: 24? Did you see it well [pointing at another learner]?

Lr3: 45

Class: No. [T asks another learner.]

Lr4: 44

T: Is he correct?

Class: Yes

T: Let's count and check starting from 41, do you see it?  
[The teacher pointed to the numbers on the 100 square from 41, pointing to each number as the class counted from 41– 44.]

This was followed by an episode where the number 26 was hidden and asked for, with the right answer given by a learner. The next questions asked were: 'What number comes after 26?' and 'What number comes before 26?' – with

both of these numbers visible before and after the small blank square on the 100 square on the board.

### Analysis of Excerpt 3

A key feature of Ms. S' work within this excerpt was her recurring reference to 'seeing' the hidden number. Given this recurrence, we are unsure as to whether the object of the activity was simply to recall the hidden number by remembering what it looked like before it disappeared from view or to use the structure of the square to deduce the hidden number. Her mediated actions suggest recall more often than deduction using the 10-based structure of the 100 square, although the episode involving 44 suggests that the teacher did have some awareness of the sequence of number. Even within this one section, the explanation that is used to 'check' whether 44 is correct was based on reciting the number list from 41 – again, an everyday rather than a specialised form of expression that makes no explicit reference to the ways in which rows of a 100 square are structured and how looking down the columns is an effective strategy: there was no reference at all to the column structure of 100 squares, which, like the abacus, is a representation that embodies the 10-based structure. Modelling of how to find the number that would use the row and column structure – 44 is in the row that contains all the numbers in the 'forties' and the column that contains all the numbers with 4 in the units – was visibly absent. Learners' attempts to identify 44 – with 34 and 24 offered – suggest that for some children the vertical patterns in the square had begun to have some ideality but this was not picked up and worked with by the teacher. Within the highly partial take up of the base-ten numerical structure, we see materiality once again within mediation rather than ideality.

## Conclusions

In our analysis we have brought together the theoretical concepts associated with mediated action: the notion of an agent working with an artifact to model the ideal in the expectation that learners might appropriate actions and come to achieve a desired object. Within each of our excerpts we argue that our analyses of the teacher-artifact mediation observed was likely to restrict any tool use to unit counting. While this may be located in an individualised view of limitations in individual teachers' PCK we also want to argue that there is a

cultural dimension to these limitations and return to Wertsch's work to examine this.

Vygotsky's (1981) assumption about the fundamental role of mediating tools (psychological as well as physical tools) in mental function was that:

by being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act (Vygotsky, 1981, p.137).

Here, the presence and use of artifacts is viewed as naturally leading to their appropriation. Wertsch's position (1995) differs somewhat from Vygotsky's by expanding the focus on tools into the sociocultural setting in which they are located, and points out the take-up of artifacts in the dialectic between tools and thinking as reliant on a further condition:

the mediational means that shape mental functioning and action more generally are inherent aspects of, and hence serve as indexes of, a sociocultural setting (p. 64).

In our empirical problem this second condition is problematic. The push to get resources into schools via the FFL policy suggests that (in poorer schools at least) these resources were not previously a part of the socio-cultural setting, and thus we cannot assume that the mathematical structure of a resource that might be apparent within certain sociocultural settings (including those of the policy makers) is also a feature of the sociocultural setting of these schools. We have shown that material, rather than ideal, use of the artifact was a common feature across all three of the empirical excerpts presented in this paper. The almost complete lack of reference or use of the 10-based structure of both the abacus and the 100 square raises questions relating to these teachers' awareness of this structure from both a content knowledge and pedagogical content knowledge perspective. But we suggest that this is a consequence of absence in the sociocultural setting, rather than deficit in the teachers' knowledge.

Our data, collected in early 2011 in the context of FFL with its explicit attention to resource provision suggests that a wider range of resources (including some that embody features related to more symbolic representations of number such as 100 squares and abacuses) are making their way into schools. Our analysis shows though that provision of artifacts that support more abstract ways of working with number is insufficient in improving teaching and learning. Understanding of the ways in which early

counting progresses into mental procedures – expressed succinctly by Askew and Brown (2003) as: ‘count all, count on from the first number, count on from the larger number, use known facts and derive number facts’ (p.6), and how progressive counting leads into more abstract number concepts (Gelman and Gallistel, 1986) – would appear to be required in the socio-cultural setting, prior to being able to recognise the potential usefulness of these presences in artifacts that model these reified structures.

Research tells us that carefully structured experiences with children focused on developing more compressed operational understandings and the more abstract notions of number that these are founded on and further develop are possible (Askew, Bibby, and Brown, 2001). Our future work is to develop structured experiences for teachers that can work at this content knowledge level whilst working at the pedagogical action level as well. Our analysis suggests that this is needed in order to support the development of mediated actions that allow for the ‘specialised’ mediation required to capitalise on the improving resource situation in ways that have impact at the level of children’s learning.

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# Summative assessment of student teaching: a proposed approach for quantifying practice

Lee Rusznyak

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## Abstract

A pass mark in a teaching practice module is a convenient way for universities to signal confidence in students' beginning teaching competence. However, assigning marks for teaching competence is a fraught undertaking if marks are to be standardised across different assessors *and* reflect teaching as a complex, coherent practice. This paper analyses reports written by university tutors justifying the marks awarded to a cohort of final-year student teachers for their teaching practice. The analysis shows that marks reflect an interplay between the students' pedagogical thinking (evident in the rationale for their lesson design and written and verbal reflections on their teaching), and their ability to deliver lessons effectively (from direct observation of their teaching). This finding prompted the development of a Summative Teaching Practice Assessment Rubric which considers both the cognitive and performance dimensions of student teaching. It potentially enables a more coherent, holistic summative assessment of student teaching than had been possible using lists of isolated criteria or general impressions of competence.

## Introduction

Higher education institutions are responsible for ensuring that newly-qualified teachers are able to assume responsibility for classroom teaching at the start of their careers. A credit or 'pass' mark in a teaching practice course is a convenient way for institutions to signify their confidence in qualifying students' readiness to begin teaching. Observations of student teaching together with other sources of supporting evidence (such as students' rationale for their lesson design, reflections, devised learning and assessment tasks and their contributions during post-observation discussions) contribute to a holistic profile of student teaching on which assessment can be based (Fraser, Killen and Nieman, 2005; Darling-Hammond and Snyder, 2000). Summative assessment instruments are supposed to guide large numbers of university tutors and supervising teachers as they undertake these observations and make

high-stakes judgments about students' teaching competence or incompetence. This study was prompted by dissatisfaction with a particular guideline for the summative assessment of student teaching previously used by staff at the Wits School of Education (see Appendix A). That guideline provided university tutors with a list of twelve criteria for assessment, some of which were inherited from documents used before the merger between the Johannesburg College of Education and the Wits Faculty of Education, while others were derived from the Exit Level Outcomes stipulated by the Norms and Standards for Educators (Department of Education, 2000). The guideline provided no indication of how to translate students' teaching competence into a single mark. Some university tutors wrote an open-ended report in which they commented on observed teaching and supporting evidence to justify the impression mark that they had awarded. This approach enabled university tutors to consider different forms of evidence related to students' teaching in a coherent and integrated way, but negated the possibility of a reliable approach that could be easily standardised across the various teaching subjects and phases. Other university tutors determined a final mark by assigning a numerical rating to each criterion and obtaining a cumulative total. The use of marks against checklists provided clearly visible criteria, but also assumed that every criterion had equal value and could be considered as an isolated competence. This approach might indeed be suited to technical training (which applies facts, rules and procedures to a range of predictable situations), but it is far from an ideal way to assess practices that involve professional judgment (Martin and Cloke, 2000; Coll, Taylor and Grainger, 2002).

Dissatisfaction with the limitations of both these approaches prompted an investigation into what university tutors recognise as distinctive student teaching, and a qualitative analysis of the reports written to justify marks awarded. The findings informed the construction of a different kind of *Summative Teaching Practice Assessment Rubric* (Appendix C). It prompts university tutors and mentor teachers to consider various forms of evidence to assess both the performance *and* the cognitive dimensions of a student's teaching and suggests a standardised mark range that takes both these dimensions into account. The structure of this rubric potentially offers teacher educators a more principled and coherent approach for the summative assessment of student teaching than is possible through approaches using lists of criteria or overall impressions of competence.

## Summative assessment of student teaching

Whereas some higher education institutions offering teacher education in South Africa allocate a 'credit' for students' final session of practical teaching, others assign a mark (Reddy, Menkveld and Bitzer, 2008). Where a non-quantitative 'credit' is awarded for student teaching, teacher educators need only establish that the student teacher is *not incompetent* during their final session of teaching practice. The allocation of a mark for teaching competence has some merit, as the wide range of marks on the percentage scale can be used to acknowledge excellence in student teaching by awarding a distinctive mark, or signalling a minimal level of competence by a pass mark of 50%. The assigning of marks for student teaching provides for the comparative profiling of results for the purpose of issuing awards, especially when considering students teaching over different subjects and/or phases. However, the multifaceted nature of the evidence used to determine the quality of student teaching, and the complexities of teaching itself makes it difficult to represent teaching competence in a single numerical value (Uhlenbeck, Verloop and Beijaard, 2002; Darling-Hammond and Snyder, 2000).

The summative assessment of student teaching has been described as "contentious and complex" (Reddy, Menkveld and Bitzer, 2008, p.155). The Minimum Requirements for Teacher Education Qualifications stipulates that teaching practice should be both supervised and assessed (Department of Higher Education and Training, 2011). However, a recent review of South African initial teacher education programmes raised concern about the "design, monitoring and assessment of teaching practice", noting particularly a pervasive "lack of common understanding of . . . assessment rubrics" (Council of Higher Education, 2010, p.94). Summative assessment instruments should ideally make explicit the principled grounds upon which marks are awarded to different stakeholders: university tutors (who need to be accountable for the judgments they make); student teachers (who could benefit from understanding how excellence within a practice is recognised); the wider teaching profession (who participate in mentoring and assessment of student teachers); the State (as accreditors, policy-makers and future employers of student teachers). It is not surprising then that the summative assessment of student teaching has been identified as "one of the major challenges facing practicum supervisors and teacher educators in general" (Reddy *et al.*, 2008, p.146).

## Competence in teaching

Nationally and internationally there is a well-documented tendency for school-based mentors and university tutors to focus their attention on the performance of teaching without due consideration of the cognitive thinking underlying it (e.g. Zanting, Verloop and Vermunt, 2001; Roelofs and Sanders, 2007; Reddy *et al.*, 2008). Shulman's (1987) Model of Pedagogical Reasoning and Action provides a potentially useful framework for understanding teaching with consideration of the "intellectual basis for teaching performance, rather than on behaviour alone" (p.107). He insists that teachers need first to *comprehend* the content knowledge or text to be taught before a lesson takes place. Teachers' understanding of key concepts enables them to *transform their understanding* into appropriate representations understandable to classes of diverse learners. Transforming the content into an accessible form thus culminates in a pedagogically-reasoned "plan, or set of strategies, to present a lesson, unit or course" in which teachers take pedagogical decisions based on a simultaneous consideration of the demands of the content to be taught, the needs of their learners and the possibilities within their teaching contexts (p.104). During the interactive stage of teaching, which Shulman calls *Instruction*, teachers and learners are simultaneously involved with the concept, text or topic to be learnt. He refers to "observable forms of classroom teaching", in which the prospective plan is enacted and adjusted in response to the learning environment created during a lesson (p.101). It includes classroom management as well as presentation of content, interaction with learners, and assigning of learning and assessment tasks. In order for a teacher to act flexibly and responsively to the ever-changing dynamics within a lesson, the teacher must be continually thinking about what is happening and how learners are engaging with the concepts. Shulman defines reflection as "the set of processes through which a professional learns from experience" by reviewing the lesson in relation to the purpose that the teaching intended to achieve ( p.106). It often takes place in the post-active phase following a lesson when a teacher "looks back at the teaching and learning that has occurred, and reconstructs, re-enacts, and/or recaptures the events, the emotions and the accomplishments" (p.106). As a result of 'reasoned' teaching, the teacher comes to a "new comprehension" of the "purposes and of the subjects to be taught, and also of the [learners] and of processes of pedagogy themselves" (p.106). In all these processes Shulman shows how the visible actions of classroom teaching are underpinned by a knowledge base that supports the making of considered pedagogical choices. While there are

many calls for student teaching to be assessed with due consideration of cognitive and performance dimensions of the practice, there is little in the literature on how this call might be enacted in practice.

Initial teacher education programmes should enable students to develop a beginning repertoire of teaching knowledge and skills that lay a basis from which they can continue to learn from their practice and eventually develop expertise (Feiman-Nemser, 2001). Studies on the practices of expert teachers have identified attributes of excellence in teaching that enable powerful learning (e.g. Berliner, 1994; Hattie, 2003; Hayes, Mills, Christie and Lingard, 2006). Findings in these studies suggest that *expert* teachers (as compared to those that are *accomplished* or merely *experienced*) have efficient automated routines; engage learners in learning activities that build deep knowledge and understanding; provide relevant feedback, use appropriate representations of concepts, are responsive to learners needs, and create safe learning environments. Some attributes refer to the thinking that teachers do before and after the lessons they teach (such as their capacity to devise appropriate representations of the content they teach) and others refer to how they manage the learning process during the course of the contact time with the learners (such as how they use routines to maximise teaching time). These studies are exceptionally valuable in guiding teacher educators attention to those aspects of students' developing practice that make the most impact on learning (both in coursework and in their formative feedback on students' attempts at teaching). While students should be developing these vital aspects of their practice, the attributes of expert teaching should not be used as a checklist for rating the competence of teachers (Hattie, 2003). Furthermore, it is unrealistic to expect that students will have a fully developed expert practice by the time they qualify (Feiman-Nemser, 2001). It is therefore important to identify the attributes of competent student teaching and incompetence that characterises student teaching. Reynolds (1992, p.1) produces a list that describes "what beginning teachers should know and be able to do" by the time they qualify. Some criteria she lists refer to the cognitive dimension of teaching (e.g. students' knowledge of the subjects they teach; their consideration of the needs of learners; their ability to make appropriate pedagogical choices, their capacity to plan coherent lessons and their ability to reflect on their teaching). Other criteria propose what beginning teachers should be able to do during the lessons they teach (such as relate well to learners; establish and maintain routines; construct a conducive learning environment and assess learning). These capacities can be observed and could be seen to constitute the *performance dimension* of competent student teaching. While the findings and

criteria from these studies are valuable in designing descriptors for formative assessment rubrics that prioritise the kinds of knowledge, skills and dispositions that student teachers should develop during their initial teacher education programmes, they do not offer a way of using the criteria to translate students' practice into a numerical mark in a standardised and coherent way. In contrast, Raths and Lyman (2003) describe *incompetence in student teaching* as "acts of commission or omission on the part of the [student] teacher that interfere with the learning processes of learners, or that fail to advance them" (p.211). They propose that these acts of commission/omission could include a lack of understanding of the subject matter content; inability to incorporate feedback from previous lessons into subsequent planning; inability to relate to learners, and/or not engaging learners in high-quality active learning. While their comprehensive list is potentially useful for exploring the grounds on which student teaching can be deemed 'not yet competent' for qualification, their guidelines do nothing to distinguish between different levels of developing but nonetheless competent student teaching. Little is known about the attributes of the teaching observed from the most accomplished of student teachers at the end of their initial teacher education programmes.

## Methodology

Student teachers at the Wits School of Education undertake periodic sessions of practical teaching, mentored by a supervising teacher and a university tutor. University tutors undertake several lesson observations of each student allocated to them and examine other evidence of thinking and planning (such as the student's lesson preparation and reflective journal). After each observed lesson, they meet the student teacher to reflect on issues of the teaching and learning arising during the lesson. In the first three years of their BEd degree students are thus mentored, and their ongoing progress is *formatively assessed* with the intention of providing feedback that promotes their professional development (Rusznyak, 2011). A different approach needs to be used for final year student teachers who are *summatively assessed* by university tutors to verify their teaching competence prior to qualifying.

The search for a more systematic, principled approach to summatively assessing student teaching holistically required a two-phase qualitative investigation. First it was necessary to understand the grounds on which

experienced university tutors award distinctive marks for teaching practice, and how these grounds related to the 12 criteria contained in the guideline that was used at the time. Sixteen university tutors accepted an invitation to participate in a focus group discussion. All had experience in observing and mentoring student teachers, and assessing their teaching within their particular phase and/or subject specialisation. The group was asked to ‘describe student teaching that you consider to be worthy of a distinctive mark (above 75%)’. Detailed notes were made during the discussion, and consensus emerged around thirteen attributes of distinctive teaching that were suggested (See Appendix B).

The empirical data for the second phase of this investigation were the reports written to justify the marks awarded for a cohort of 46 final-year BEd students who specialised in Intermediate/Senior phase teaching. The reports were written by fourteen university tutors appointed to assess the cohort of student teachers. All of these tutors had prior experience in the summative assessment of student teaching and in previous years, they had all participated in annual internal moderation meetings in which the grounds on which they had awarded marks had been discussed and adjusted where necessary. Five of these tutors had also participated in the focus group discussion about the attributes of distinctive teaching.

Shulman’s distinction between *pedagogical reasoning* and *pedagogical action* was useful in analysing the reports written by university tutors to justify their marks. Comments in each report were coded according to those that referred to the *cognitive dimension* of students’ teaching (their knowledge and understanding of the concepts taught, rationales for pedagogical choices and reflections on their teaching) and those concerned with observable *classroom performance* (such as their classroom management, interactions with learners, use of resources, execution of teaching strategies and pacing). The reports could then be clustered into four categories according to the nature of the comments university tutors made regarding the students’ pedagogical thinking and their pedagogical action. The four categories were defined as follows:

1. Reports that commended thinking and lesson delivery
2. Reports that commended student thinking but noted challenges with lesson delivery

3. Reports that commended performance but expressed concerns about conceptualisation of and/or reflection on lessons
4. Reports that expressed concern about the students' ability to think about their teaching and deliver lessons effectively

The mark profile of each category was analysed, and patterns were identified. The findings of this analysis and the literature around cognition in teaching enabled the construction of an alternative rubric for the summative assessment of student teaching. I describe the structure of the rubric and then use the report of one student, Fatima, to illustrate how the rubric suggests a mark range for her teaching given the particular strengths and weaknesses identified in her tutor's report written on his observations of her teaching and his examination of her lesson preparation.

## Limitations of this study

The data relies on the observations and interpretations of student teaching by university tutors who were appointed to assess this cohort of students. The analysis assumes that each report written is a complete account of each student's teaching. The study also assumes that the fourteen university tutors who wrote the reports possess the capacity to make appropriate professional judgments about the teaching competence of student teachers. Cochran-Smith and Lytle (1999, p.263) conceive of teacher knowledge *in practice* as "embedded in experience and in the wise action of very competent professionals". Such professional knowledge enables an "appropriate perception of what is salient in particular situations" (Morrow, 2007, p.80). Despite the limitations, the reliability of this study is significantly enhanced by the experience of the 14 university tutors and the internal moderation processes that fostered the development of shared standards for assessing students.

## Findings:

### The attributes university tutors recognise in distinctive student teaching

The focus group discussion yielded consensus on 13 such attributes (Appendix B). The 13 attributes of distinctive student teaching generated included both cognitive and performance aspects of student teaching. These were then compared with the criteria competence of the guidelines (Appendix A). It was immediately apparent that the official criteria emphasised the *performance dimensions* of student teaching, whereas the attributes of distinctive student teaching focused more on the underpinning *understanding and thinking* that went into making appropriate pedagogical choices, as evident in Table 1. This finding suggests that a much higher level of cognition characterises student teaching that is recognised as being distinctive.

**Table 1: Comparison of cognitive and performance dimensions of the criteria for assessing student teaching competence and with attributes of distinctive student teaching**

|                              | Pedagogical reasoning and action | Given criteria for assessing competence (from Appendix A)   | Attributes of distinctive teaching (from Appendix B)   |
|------------------------------|----------------------------------|---|--|
| <b>COGNITIVE DIMENSION</b>   | Comprehension                    | <ul style="list-style-type: none"> <li>Degree of knowledge &amp; insight into relevant subjects</li> </ul>  | <ul style="list-style-type: none"> <li>Thorough knowledge of topics taught and how they relate to rest of curriculum</li> <li>Ability to distil key concepts from the detail</li> <li>Making learning relevant and current</li> </ul>  |
|                              | Transformation                   | <ul style="list-style-type: none"> <li>Planning, preparation and integration of units of work</li> <li>Development of support materials</li> <li>Design of learning tasks and assessment tasks for assessment of learner development</li> <li>Variety and appropriateness of teaching strategies</li> </ul>   | <p>Lessons that develop a systematic learning process</p> <ul style="list-style-type: none"> <li>Sense how this learning links with previous and future lessons</li> <li>Choice of innovative teaching strategies conceptually appropriate to lesson content.</li> <li>Catering for diverse needs of learners</li> </ul> |
|                              | Instruction                      | Not applicable  | <ul style="list-style-type: none"> <li>Flexible and responsive during the lesson</li> <li>Meaningful engagement with learners</li> </ul>   |
|                              | Reflection                       | Not applicable  | <ul style="list-style-type: none"> <li>Rigorous &amp; insightful reflection</li> </ul>   |
| <b>PERFORMANCE DIMENSION</b> | Instruction                      | <ul style="list-style-type: none"> <li>Effective use of support materials</li> <li>Ability to motivate, arouse and maintain interest of learners</li> <li>Ability to communicate</li> <li>Assessment of learner development</li> <li>Effectiveness of class discipline strategies</li> <li>Classroom management</li> <li>Quality of relationship with learners</li> </ul> | <ul style="list-style-type: none"> <li>Choice of innovative teaching strategies conceptually appropriate to lesson content.</li> <li>Meaningful, responsive engagement with learners</li> <li>Flexible and responsive during the lesson</li> <li>Creating a safe learning environment</li> </ul>                         |

Both the criteria for competence and the attributes of distinctive teaching were organised as lists. By their very nature lists do not necessarily show any conceptual hierarchy between items on the list or how the items listed interact with one another. To understand the grounds on which assessment decisions are made as aspects of a coherent practice rather than discrete elements on a list, it was necessary to analyse the university tutors' reports that justified the marks they awarded for student teaching.

## Analysis of reports justifying marks awarded for student teaching

The open-ended reports that justified marks could be classified into four broad categories. In the first category are reports that commended both students' knowledge and ability to think pedagogically, and their effective execution of lessons. Reports in the second category commended students' competence in interaction with learners, but expressed concerns about their ability to conceptualise lessons and/or reflect on their lessons. In the third category, reports contained comments that commended students for their ability to think about their teaching but noted concerns about their ability to deliver their lessons effectively. Fourthly are those where concern was noted regarding both students' ability to think about their teaching and to deliver lessons competently. I will now briefly describe each of the four categories.

### Category 1: Reports that commended thinking and lesson delivery

Reports in this category commended students for their subject knowledge, thoughtful pedagogical choices in the planning stages and their ability to deliver their planned lessons effectively. A tutor, for example, describes how a student *maintains a sound working environment in that her learners were always busy and productive*. Although not a criterion on the guidelines, the ability of students to reflect on and understand their teaching was noteworthy in reports in this category. Hence, for example, comments like, *reflective notes at the end of lessons were insightful with regard to the effectiveness of her teaching in relation to the development of her learners*. The ability of the student to be responsive to learners' needs during their lessons was also frequently emphasised. Thus, a tutor commends the way a student was able to notice, adjust and refocus her lesson: *Once [the student] realized that the*

*material contained too much information for some learners, she revised and shortened the texts and tasks.* Having the flexibility to respond to unpredictable classroom dynamics in a manner that maximises learning opportunities was commended in these reports.

In several reports, university tutors had commended students' thinking and lesson delivery but had included qualifiers, such as 'mostly', 'sometimes' and 'often' which suggests that although their teaching had been found to be competent overall, this level of competence was not consistently maintained.

### Category 2: Reports that commended student thinking but noted challenges with lesson delivery

Commendation here was for thorough and thoughtful lesson planning, such as *she showed excellent insight and knowledge in different subjects, and she was well prepared in advance.* However, reports contained expressions of concern about the students' ability effectively to convert their lesson plans into effective lessons. One such read: *At times, very well prepared lessons with sound educational intentions came to little because effective discipline could not be maintained.* In some cases, such 'challenges' were exacerbated by contextual factors, such as: *having to move from classroom to classroom contributed to the difficulties she experienced with the class.* In other cases students' difficulties were attributed to their developing teaching skills, such as: *She still needs to develop the way she gives instructions, and limit the interventions in the lesson after she has given her instructions. While it is important to give deadlines and deal with common queries, it was often at the expense of the flow of the lesson.*

### Category 3: Reports that commended performance but expressed concerns about conceptualisation of and/or reflection on lessons

These reports describe students' strong classroom personalities with effective interaction with learners. For example, a tutor describes a student who *seems to relish the role of teacher and works with calm confidence in the classroom, also with real empathy for learners;* yet, despite this positive attribute, the student needed to *demonstrate the ability to do more than act as a good babysitter for the teacher.* Reports expressed concern about the conceptual

depth of lessons, the quality of thought reflected in the lesson planning, and occasionally their ability to reflect on the teaching and learning in their lessons. Several tutors commented on lessons that had been enjoyable for learners, but lamented *busyness rather than purpose*. Thus, a tutor acknowledges that the student is *competent enough to keep learners quiet and busy* but challenges the student to demonstrate *well thought out, conceptually strong teaching*. Several of these reports suggested that the student relied more on their personalities than on an intentional intervention to enable new learning.

With pressure from the tutor to attend to their planning, many students in this category improved dramatically in the quality of their teaching over the practicum. One report describes how a student *discovered that preparation not clearly thought out results in disastrous, inconclusive lessons. These problems have now led to more methodological, systematic (and successful) lesson planning*. This category includes potentially capable students who either underestimate (or try to avoid) the amount of thinking required for coherent lesson planning.

**Category 4: Reports that expressed concern about the students' ability to think about their teaching and deliver lessons effectively**

Here, reports suggested that students' understanding of content knowledge caused great concern. One tutor was concerned about a particular student's ability to understand and organise content knowledge, noting that the student *kept straying off the topic and often explanations were not coherent*. Another expressed concern about a student's teaching competence despite her caring disposition: *Without thorough planning and preparation, she is insecure in the classroom, for she is struggling to think of what to say and what to do next. In future, she needs to ensure that all this is in place well before she starts her lessons. Furthermore, she must do more in-depth research: what is provided in the textbook is insufficient. She clearly cares for learners and learners respond to her, but at times this interaction is not directed to learning*. The report suggests the challenges in lesson delivery are sometimes attributable to weaknesses in the student's understanding of the content and planning.

**Table 2: The mark distribution within each category of reports**

|   | < 50%    | 50% – 54% | 55% – 59% | 60% – 64% | 65% – 69% | 70% – 74% | 75%+      | Number of reports in each category |
|---|----------|-----------|-----------|-----------|-----------|-----------|-----------|------------------------------------|
| 1. Number of reports that commended thinking and lesson delivery  | 0        | 0         | 0         | 2         | 7         | 4         | 18        | <b>31</b>                          |
| 2. Number of reports that commended student thinking but noted challenges with lesson delivery  | 0        | 0         | 0         | 2         | 1         | 3         | 0         | <b>6</b>                           |
| 3. Number of reports\ that commended performance but expressed concerns about conceptualisation of and/or reflection on lessons       | 0        | 0         | 0         | 1         | 2         | 3         | 0         | <b>6</b>                           |
| 4. Number of reports that expressed concern about the students' ability to think about their teaching and deliver lessons effectively | 1        | 2         | 0         | 0         | 0         | 0         | 0         | <b>3</b>                           |
| <b>Number of students whose mark fell in the mark range</b>   | <b>1</b> | <b>2</b>  | <b>0</b>  | <b>5</b>  | <b>10</b> | <b>10</b> | <b>18</b> | <b>46</b>                          |

The reports of all students awarded distinctive marks fell within in Category 1 (although not all students in this category obtained distinctions). Reports in Category 1 who obtained marks of below 70% were those in which university tutors used qualifiers to denote some reservations about consistency. Students deemed minimally competent or who failed their teaching practice were all located in the Category 4. The grounds for awarding distinctions and marks of less than 60% were fairly consistent. By contrast, reports for students ranging from 60% to 74% fell variously within Categories 1, 2 and 3. In all the

categories justifications for marks awarded were made on the grounds of how the students' thinking about and understanding of their teaching related to their ability to realise their intentions for learning in the lessons they delivered.

Contrary to claims in the literature that cognitive dimensions of teaching were frequently ignored, the analysis of the open-ended reports justifying marks showed that, in assigning a mark, tutors in this study considered both the students' thinking and also their ability effectively to create productive learning experiences during their lessons. Furthermore, analysis of the range of marks awarded to the four categories of reports suggested that strengths and weaknesses in students' teaching thinking and classroom action combine in ways that reflect in patterns of marks awarded across the cohort of students. This finding provided a way for the design of a rubric in the form of a two-dimensional grid rather than a linear list (see Appendix C).

## Design of a different kind of summative teaching practice assessment rubric

The revised rubric for the summative assessment of student teaching describes a continuum consisting of five levels of *teacher understanding and thinking* (across the top row) and a continuum consisting of five levels of *teaching performance* (along the far left column). Assessment then takes the form of identifying the appropriate level of students' *teacher understanding and thinking*, and an appropriate level of their *teaching performance*. Plotting the horizontal to the vertical, identifies a cell that suggests a range of appropriate marks. Perhaps the chief merit is that the suggested range for different combinations of strengths and weaknesses is consistently accessible for all university tutors and supervising teachers assessing final year student teachers' teaching competence.

### Assessing teacher thinking

The dimension of *teacher thinking* refers to those aspects of a teaching practice that are not directly observable during classroom action, but which nonetheless require subject and pedagogical knowledge and reasoning prior to and after the lesson itself. In accessing and assessing this, university tutors

draw on several forms of evidence. Firstly, students' lesson plans are not merely a record of their intended actions, but are structured to provide a rationale for their lesson design (Rusznayak and Walton, 2011). The tutor can thus assess the degree of thoughtfulness in the planning. Secondly, the students' reflective journals, which should be kept up-to-date throughout the practicum provide evidence of how the students reflect upon the teaching and learning during lessons they teach. Thirdly, after every lesson observed, students are expected to meet with the university tutor/mentor teacher to discuss their lesson. The assessor has opportunities during these post-observation discussions to probe the student's perception about their teaching and the learning that took place. The kinds of teacher thinking that are evident from the above sources includes the students' understanding of content knowledge, thoughtfulness of their lesson preparation, their ability to reflect on their practice, and the degree of insight and innovation they bring to the design of their lessons. The separation of 'teacher thinking' from 'teacher action' in the rubric does not imply that no teacher thinking takes place during the lesson itself, but in order to assess this dimension, the university tutor looks for evidence in sources other than the direct observation of student teaching.

### Assessing teacher action

The dimension of *teacher action* refers to those aspects of teaching that are directly observable during the time in which students are in contact with a class of learners during the lessons they teach. The kinds of observable aspects of students' performance includes the way in which they interact with learners, their ability to communicate, their classroom management and their responsiveness to learners and the learning dynamics during the lesson. It includes students' ability to work productively with learner responses and managing the learning processes. In order to identify an appropriate range for a student's mark, the assessor is required to undertake several classroom observations. While this represents only a fraction of the total teaching time, a joint assessment by a university tutor and a diligent mentor teacher would be able to build a more complete picture.

## Using the rubric

The rubric suggests a mark range in a cell where the two dimensions of a students' teaching practice intersect. It still reduces practice to a single mark, but this mark represents an assessment of teaching as a practice that relies on both competent pedagogical thinking and competent pedagogical action. By describing a continuum of five levels each of teacher thinking and teacher action, the rubric is able to accommodate the large variations observed in the student teaching in the analysis of the reports and suggest an appropriate range of marks. The rubric also indicates the grounds for an assessment of student teaching that is not yet competent for certification. Students who persistently misunderstand the content they teach; put little thought into their preparation; behave unprofessionally, or are unable to execute their lessons effectively are not yet ready to assume independent responsibility for a class of learners. In these cases, the rubric recommends a mark of below 50%. In order to obtain a distinctive mark, students would need to demonstrate both the capacity to think insightfully about their teaching, and to create productive learning opportunities in the lessons they teach.

## Using the summative teaching practice assessment rubric to assign marks for student teaching

The following open-ended report was written by a university tutor to justify the mark awarded to a student teacher in her final year of study. There is evidence that the university tutor referred to the list of criteria (Appendix A) when writing the report to justify the awarded mark of 68%. The report includes references to the student's lesson planning (criterion 2); support materials (criterion 3); her ability to motivate, capture and sustain the interests of learners (criterion 5); class discipline (criterion 7); ability to communicate (criterion 9) and her relationship with learners (criterion 10).

*Fatima has a well-organised preparation file and her lesson planning is very good and this is her strength. She is respectful towards her supervisors and attempts to implement suggestions and guidance provided by them. She usually saw to it that she had all the support materials on hand to ensure the success of her lessons. She displayed satisfactory knowledge of the underpinning concepts of her lessons. Good written planning and preparation are not always guarantees of success in teaching. The effective management of the learning experience is important, including pacing, discipline and enough by the way of content and activity to maintain the interest of learners. Good intentions were destroyed when she let the pacing of the lesson to slip, because with that the discipline slipped too. She*

*must work at maintaining the interest of her class by avoiding lengthy explanations and then start an activity before learners become fidgety. Fatima obviously enjoys being in the classroom, but must pay attention to her lesson delivery and her relationship with learners.*

Fatima's strength lies in her thoughtful and careful planning but the report suggests that she experiences difficulty in translating her planning into effective learning opportunities. The report written about her teaching is typical of those in Category 2. The report that Fatima demonstrated Teacher Thinking at Level 4 and Teacher Action at Level 2. The rubric suggests that a mark of between 60–64% would be appropriate for Fatima (see Appendix C). The rubric suggests a slightly lower than the mark than that of the tutor awarded for an overall impression of Fatima's teaching competence.

## Limitations and implications

Several refinements to the rubric have been over the past two years as we first piloted its use and then adopted it as the institution's formal instrument for Summative Teaching Practice Assessment. As we continue to deliberate what characterises excellence in student teaching, various adjustments have been made to the level descriptors in both the Teacher thinking and Teacher action dimensions. It is possible that the teaching of a particular student cannot be reliably placed along the continua of either (or both) of the dimensions of teacher thinking and teacher action. For example, one might come across a student whose planning demonstrates thoughtfulness, but whose reflections on teaching and learning are not at the same depth. In such cases, it might not be possible to pinpoint just one cell in the grid to determine an appropriate mark range. In such cases, the university tutors would need to consider a mark within the wider mark range indicated over two cells. Despite these limitations, we find there are now more transparent and accessible grounds upon which we can quantify the competence of student teaching at the end of their initial teacher education.

## Conclusion

Any summative assessment rubric that assigns a mark to teaching carries the hazards of reducing a complex practice to a single mark. We continue to debate whether a mark against a teaching practice course is an appropriate way to reflect the university's confidence in students' teaching competence.

However, as no satisfactory alternative exists at present, then summative assessment rubrics used should reflect aspects of the complexity inherent in teaching. The revised Summative Teaching Practice rubric prompts university teachers and supervising teachers to consider a wide range of evidence when making decisions about student teaching competence at the end of their initial teacher education. It offers teacher educators an approach that gives due attention to both the *cognitive* and *performance* dimensions of student teaching in more principled and coherent way than has been possible with the use of lists of criteria or resorting to awarding marks based on a general impression of competence. The simultaneous consideration of these two dimensions offers possibilities for making summative assessment of students' teaching competence more reliable and more explicit to university tutors assessing the students, student teachers and the wider teaching profession.

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**Appendix A: Criteria to guide the summative assessment of student teaching**

1. Knowledge and understanding of subject/s.
2. Planning, preparation and integration of units of work
3. Development and use of support materials
4. Variety and appropriateness of teaching strategies
5. Ability to motivate, arouse and maintain interest
6. Assessment of learner development
7. Class discipline
8. Classroom management
9. Ability to communicate: instructions; explanations, descriptions, questions
10. Quality of relationship with learners
11. Professionalism
12. Relationship with teachers and school organization

(Taken from: Guidelines for Summative Assessment of Teaching Practice, abandoned in 2008)

**Appendix B: Thirteen attributes of ‘distinctive’ student teaching**

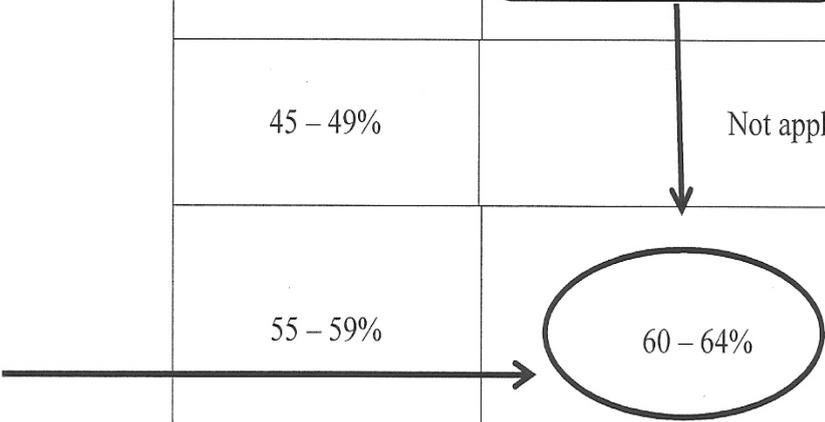
- Thorough knowledge of topics taught – and how they relate to other areas of the syllabus/learners’ lives
- Ability to distil key concepts/key issues from the detail
- Makes learning relevant and current
- Well-conceptualized lessons that systematically develop a learning process
- Innovative/creative use of teaching strategies conceptually appropriate to the lesson’s content
- Sense of larger picture – forward planning and also how this learning links with previous and future material
- Rigorous, insightful reflection – shows deep understanding of their own teaching
- Pre-empt possible misconceptions; designs tasks to expose how learners think/understand a concept
- Creates a safe learning environment for learners
- Caters for different needs/ability levels of learners
- Flexible and responsive to the dynamics within the lesson
- Meaningful, responsive engagement with learners
- Works well within the school environment

(Generated during a Teaching Practice Committee focus group discussion, Wits School of Education)

**Appendix C: Summative teaching practice assessment rubric**

|  |  |  |  |
|--|--|--|--|
| Student name:<br><b>Fatima</b>   |  | <b>Teacher understanding and thinking:</b>   |  |
| Teacher understanding and thinking level: <b>4</b><br><br>Teacher Action level: <b>2</b><br><br><b>Recommended MARK:<br/>63%</b> |  | 1: Pervasive misunderstanding of content knowledge; Little or no formal thought to the design of a learning process; reflective journal shows little engagement with issues around teaching and learning | 2: Sometimes inadequate content knowledge; Conceptualisation of learning processes is often largely limited to what has been provided to the student; limited reflection on own teaching |
| <b>Teacher action: Evidence from direct observation</b>  | 1: Lessons are often not executed effectively, so little meaningful learning takes place and / or unprofessional conduct   | 44% or less  |  |
|  | 2: Student mostly able to capture initial attention of learners, but struggles to maintain interest and momentum throughout the lesson. Some worthwhile learning takes place. Satisfactory professionalism | 45 – 49%   | 50 – 54%   |
|  | 3: Confident lesson delivery and responsive to queries of learners. Awareness of learner understanding. Satisfactory professionalism   | Not applicable   | 55 – 59%   |
|  | 4: Strong teaching performance in which learning is mediated effectively; Active monitoring of learner understanding; caters for different learning needs. Exemplary professionalism                       | Not applicable   | 60 – 64%   |
|  | 5: Responds flexibly to classroom dynamics; exceptional responsiveness to diverse learning needs; creates safe, productive learning environment; Probes learner understanding; Exemplary professionalism   | Not applicable   | Not Applicable   |

| Evidence from rationale for lesson plan, Journal for Reflection and post-observation reflective discussions  |   |  |
|--|---|--|
| 3: Basic but accurate understanding of content knowledge; Application of basic teaching methodologies to structure coherent lessons; Some meaningful reflection on lessons observed and taught | 4: Comprehensive understanding of content knowledge; Thoughtful consideration of pedagogical options and appropriate choices made; Worthwhile learning tasks with planning for formative / summative assessment; Detailed reflection on teaching and learning.. | 5: Deep insight into subject/s taught, own teaching and the needs of diverse learners; Probing reflection evident; Advance planning of consistently innovative, conceptually sound units of lessons with attention to formative/ summative assessment; |
| 45 – 49%   | Not applicable  |  |
| 55 – 59%   | 60 – 64%  | 65 – 69%   |
| 60 – 64%   | 65 – 69%  | 70 – 74%   |
| 65 – 69%   | 70 – 74%  | 75 – 79%   |
| 70 – 74%   | 75 – 79%  | 80% or over  |





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# Teacher learning through teaching and researching: the case of four teacher-researchers in a Masters programme

Sarah Bansilal

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## Abstract

There is a large body of research and on-going discussions about mathematics teachers' poor content knowledge in South Africa, with many suggesting that teachers need more opportunities to increase their content knowledge. In this article I consider one important opportunity that does not seem to be exploited – that of teachers' learning in the classroom. By considering the learning experiences of four teacher-researchers who were enrolled for a masters degree, I explore how these teachers' mathematics teacher knowledge developed as a result of their research inquiry. The findings indicate that all four teachers have deepened their mathematical knowledge for teaching in the various domains. However, learning in the classroom is enabled by the presence of supportive and knowledgeable colleagues. If authorities want to encourage such forms of learning, then attention needs to be directed to providing intensive classroom support in order to maximise the opportunity for classroom learning.

## Introduction

There are many studies that point out the importance of teachers' knowledge in developing students' understanding (Adler, Pournara, Taylor, Thorne and Moletsane, 2009; Ball, Thames and Phelps, 2008; Kriek and Grayson, 2009; Thompson and Thompson, 1994; 1996). In South Africa numerous studies have reported that mathematics teachers struggle with understanding even school level mathematics content. However there seems to be a widespread view that the only way for teachers to improve their knowledge is for them to attend classes or workshops and be taught this knowledge that they need. An important dimension that is sometimes not recognised is that of teachers building up mathematical knowledge by observing and reflecting on the teaching and learning experiences in their classrooms. A constructivist perspective suggests that knowledge is constructed. In the case of mathematical knowledge for teaching, how, where and under what conditions can this construction occur? Recently, Zazkis and Leiken (2010) have focused

on how and what teachers learn through the process of teaching itself. In this article I look at one particular setting – that of teachers who are engaged in research in their classroom as part of their masters degree studies and I consider how the classroom has acted as a powerful learning site for this group of teachers. In this case, the research process has contributed to the construction of their mathematical knowledge for teaching by providing opportunities for the teachers to engage in detailed observations and critical reflections. In addition, the teachers’ engagement with theories and studies related to their own inquiries would have provided additional sources for the reflection, while the supervisors’ support and advice would also have facilitated their growth. The purpose of this study is to explore how aspects of their mathematics teacher knowledge developed as an outcome of their research inquiry. This study draws on Ball *et al.*’s (2008) notion of Mathematical Knowledge for Teaching (MKT) which comprises two domains of *Subject Matter for Teaching* and *Pedagogical Content Knowledge*.

Although the participants in this study are a special case, insights gained from this study can help us understand how teacher-learner interactions help develop teachers’ knowledge for teaching. Specifically the paper unpacks particular instances of learning that have been prompted by the participants’ in-depth observations and reflections around what their learners are saying and doing. Mason (2010) contends that learning through teaching is more likely when teachers observe and reflect on surprising phenomena, and this study provides examples of some of these unexpected phenomena that have prompted the teachers’ reflections.

## Literature review

One can characterise conceptions of the knowledge required for teaching mathematics as being informed primarily by an internal or external perspective Ernest (1989). An external perspective suggests that mathematics knowledge is acquired from others, while an internal perspective suggests that a person can create their own knowledge. Although it is undisputed that teachers need extensive experience in learning mathematics and learning how to participate in mathematical domains, an internal perspective suggests that valuable teacher learning can occur in the classroom, during teachers’ interactions with their learners. Shulman’s notion of PCK also makes reference to the fact that much of PCK can originate in the classrooms, when he writes “Since there are

no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas *others originate in the wisdom of practice.*" (Shulman, 1986, p.9, italics author).

The literature on teachers and teaching has identified the need for teachers to be researchers (Roth, 2007). Thompson and Thompson (1996, p.19) recommend that teacher education should focus on building up teachers as researchers, although they concede that "the level of support teachers need to do this far exceeds what most teacher enhancement and teacher education programme provides".

Roth (2007) has no doubt about the fact that teachers who research their own practice become better practitioners. For some teachers "the interest in research may begin with the interest of becoming a better teacher" (p.1). In describing his first research study undertaken as a teacher, he explains that the topic arose from observing the difficulties his students experienced and from his interest in devising strategies that could improve their understanding. Similarly in this article the teacher-researchers' inquiries began with a motivation to learn more about their learners' struggles. In the process these teacher-researchers were able to "learn at quite different levels and about different phenomena" contributing to a strengthening of their mathematics knowledge for teaching (Roth, 2007, p.261). Roth found that as a teacher-researcher, the primary beneficiaries were his school and his learners. Another benefit experienced was that there was no gap between theory and practice – there was no outside researcher telling him what to do or how to change what he was doing.

Thompson and Thompson (1994) in their study of a teacher who struggled to mediate his learners' understanding of the concept of rates found that the teacher had encapsulated the notion of rate within a language of numbers and operations. The teacher's own encapsulation of the concept undermined his effort to help the child develop a conceptual understanding of rate. They suggest that in order for teachers to teach for understanding, they must be sensitive to children's thinking during instruction and to frame their instructional activities accordingly. Thompson and Thompson (1996, p.2), write of the "critical influence of teachers' mathematical understanding on their pedagogical orientations and decisions – on their capacity to pose questions, select tasks, assess students' understanding and make curricular choices".

Based on the abovementioned study Thompson and Thompson (1994; 1996) distinguish between a conceptual and a calculational orientation to teaching mathematics where the second is characterised by “expressing oneself in the language of procedures, numbers and operations” (1996, p.17). They argue further that:

A teacher with a conceptual orientation is one whose actions are driven by: an image of a *system of ideas* and *ways of thinking* that she intends the students to develop; an image of *how these ideas* and *ways of thinking* can develop; ideas about *features of materials, activities, expositions and students' engagement with them* that can orient students' attention in productive ways. . . (p.17)

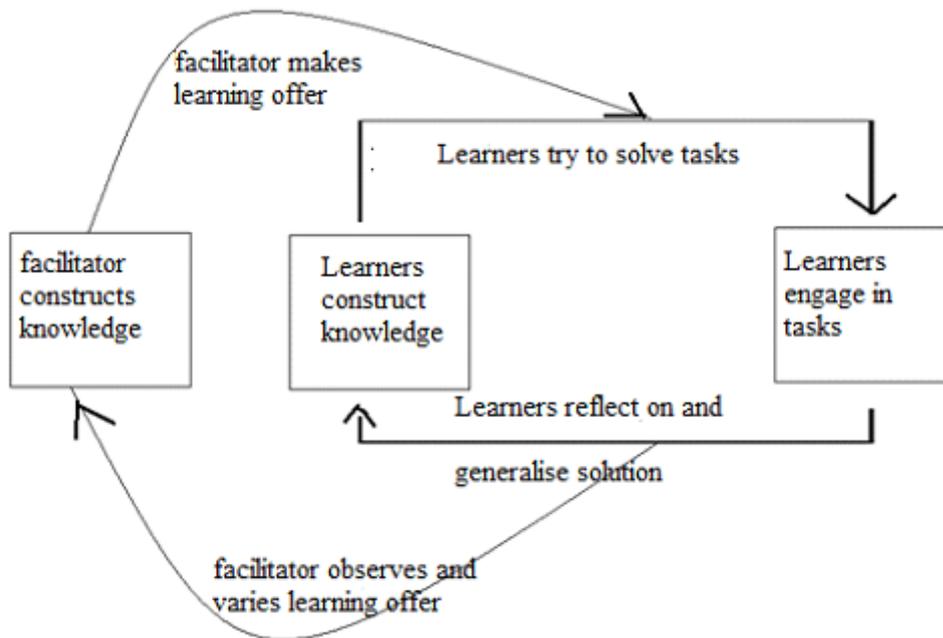
An initial calculational orientation can be shifted and refined into a conceptual orientation by engaging in sustained and reflective work with students and with mathematical ideas (Thompson and Thompson, 1996).

Maoto and Wallace (2006) described how Gerty, a teacher from the Limpopo province was able to move towards teaching for understanding, by learning how to listen. As their study progressed they found that Gerty spent more time trying to make sense of her learners learning. As Gerty tried to examine the ideas underlying her learners' confusion, she strengthened her own understanding of the mathematics she was teaching. By trying to design or adapt activities that met her learners' needs she further developed this understanding. In trying to identify the basis for some of her learners' misconceptions she had to grapple with many of the underlying mathematical ideas. In this way, her observations of, and reflections on her learners' struggles helped her improve her own understanding of what it meant to teach for her learners' understanding. It is important to note that Gerty's understanding was enhanced because of the presence of a supportive colleague with whom she could discuss her concerns, doubts and her growing awareness of teaching for learners' understanding. Similarly Davis (1997) found that the key site for the teachers' learning was the classroom itself, and the learning was facilitated by the presence of a supportive colleague.

Thompson and Thompson (1994, p.280) argue that the relationship between a teachers' and a student's ways of knowing is a reflexive one. This suggests that the teachers' way of knowing contributes to the students' development of understanding. In response, as students develop their understanding, their interactions with the teacher influences the way in which the teacher understands a concept. Somewhat related to this reflexive process is the model of Steinbring (1998, p.159) which describes how learning by both pupils and

teachers can be enhanced during a mathematics lesson. The learning of pupils and teachers, although autonomous, are linked to each other and build on each other. Steinbring emphasises the role of reflection in the learning of both pupil and teacher. Zaslavsky (2009) offers a modification of Steinbring’s model which is reproduced here.

**Figure 1: Steinbring’s model of Teaching and Learning as modified by Zaslavsky (2009, p.107)**



This figure illustrates that the teacher’s (facilitator’s) learning is “an outcome of their observations of learners’ engagements in tasks” and their reflections on learners’ work (Zaslavsky, 2009, p.107). Learners learn by engaging in a task, and also by trying to reflect on, and generalise the solutions. The teacher in turn, observes the process learners are engaged in, tries to vary the learning offers, and also reflects, which leads to his/her own learning. There are two loops of learning that are represented in this adapted model, one showing the learning by reflection of the learners and a second loop showing the learning of the teacher by reflection and observation of the process encountered by the learners. Steinbring (1998) notes:

Students’ mathematical knowledge is more personal, is bound to special exemplary contexts, and is in the process of an open development ... The mathematics teacher has to become aware of the specific epistemological status of the students’ mathematical knowledge. The teacher has to be able to diagnose and analyse students’ construction of mathematical knowledge . . . to vary the learning offers accordingly (p.159).

Central to this construction of knowledge by the teacher, is the process of reflection which facilitates the process. Thompson and Thompson (1996) suggest that teachers can come to understand a mathematical idea, in a way that enables them to teach it conceptually

through sustained and reflective work with students and with mathematical ideas – comparing their attempts to influence students’ thinking with disinterested analyses of what those students actually learn – and reflecting on both what they intended and what (they understand) they achieved (p.19).

Kraft (2002) asserts that good teacher research must centrally involve self-critical examination of those belief systems that inform and guide practice in the first place. Rosenberg (2008) has usefully summarised Hatton and Smith’s (1995) description of four levels of reflective writing as a means of thinking about teachers reflections:

1. *Descriptive writing* (writer reports on events with no reflection on them at all);
2. *Descriptive reflection* (writer provides reasons for pedagogical decisions based on personal judgement);
3. *Dialogic reflection* (writer explores the reasons for one’s pedagogical choices in the light of educational theory); and
4. *Critical reflection* (involving reasons given for decisions or events that take into account broader historical, social and political contexts).

For Brookfield (1995), critical reflection involves ‘hunting’ the assumptions that underpin our teaching practices. This process involves questioning what we take for granted and what we do in order to make our teaching lives easier, but which may not be working in the way that we assume. Paradigmatic assumptions are the hardest of all assumptions to uncover, because, as Brookfield explains,

[t]hey are the basic structuring axioms we use to order the world into fundamental categories’ and we seldom recognise them as assumptions, even after they have been pointed out to us. Instead, ‘we insist that they’re objectively valid renderings of reality, the facts as we know them to be true’ (1995, p.2).

Having looked at some research about teachers' learning in their classrooms, I will now briefly discuss the framework of teacher knowledge that underpins this study.

## Framework

The field of teacher knowledge is a vast one with many researchers generating descriptions and definitions which try to capture exactly the kind of knowledge that is needed to mediate learning in the classroom. Ball *et al.*, (2008, p.395) use the term “Mathematical Knowledge for Teaching” to refer to “the mathematical knowledge needed to carry out the work of teaching mathematics”. Their perspective is that Mathematical Knowledge for Teaching (MKT) comprises two domains which are *Subject Matter for Teaching* and *Pedagogical Content Knowledge*. Subject matter for teaching has been further divided into two subdomains of *common content knowledge* which is “mathematical knowledge and skill used in settings other than teaching” and *specialised content knowledge* i.e., “mathematics knowledge and skill unique to teaching” (Ball *et al.*, pp. 399–400). They provide examples of common content knowledge as when teachers know the work that they assign to their learners, or using terms and notation correctly, being able to recognise learners' wrong answers. Specialised content knowledge is beyond the knowledge taught to learners and includes “understanding different interpretations of the operations in ways which students need not explicitly distinguish” (p.400). Ball *et al.* also postulate a provisional third subdomain called *horizon knowledge* which “is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (p.403).

In Ball *et al.*'s model pedagogical content knowledge is divided into two further subdomains. The first is *knowledge of content and students* i.e., knowledge that permits an “interaction between specific mathematical understanding and familiarity with students and their mathematical thinking” (p.401). The second subdomain is *knowledge of content and teaching* i.e. knowledge that allows “an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning” (p.401). The authors have provisionally identified a third subdomain *knowledge of content and curriculum* which roughly coincides with Shulman's (1986, p.10) category of curricular knowledge which

is represented by the full range of programs designed for the teaching of particular subjects and topics at, a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances.

Now that the framework used to illustrate the concept of mathematical knowledge for teaching has been elucidated, the details of the methodology of the study will be discussed.

## Methodology

This is a qualitative study, utilising a case study approach in order to capture part of the storied lives of the four participant teachers, who were researching a problem they identified as part of their Master's degree studies. The participants will be referred to as teacher-researchers or as teachers, depending on which of these roles is under discussion. Data was generated from the thesis reports of the teacher-researchers, and my own field notes (as their research supervisor) drawn up during the approximately 150 hours that I spent with each of the participants during their research. The process followed in constructing the vignettes is what Polkinghorne (1995) described as narrative analysis. During the process of analysis, outcomes were roughly identified, from the data. Thereafter I went back and forth from the data to the emerging descriptions building up and identifying "thematic threads" relating to those outcomes (Polkinghorne, 1995, p.12). The happenings were then configured or 'emplotted' to take on a narrative meaning – they are understood from the perspective of their contribution and influence on the teacher-researchers' construction of mathematics knowledge for teaching. That is, each vignette is a reconstruction of events and actions that produced the particular outcomes. The second stage was to then use an "analysis of narratives" (Polkinghorne, 1995, p.12) technique to find best fit categories that were consistent with Ball *et al.*'s (2008) mathematics knowledge for teaching framework. The purpose of the analysis was to provide answers to the central research question: How does the research inquiry process contribute to the construction of mathematics teacher knowledge by the teacher- researcher?

## Results

In this section I present vignettes of each of the four teacher- researchers drawn from the data.

### Learning about learners' assessment feedback expectations

Naidoo, a Grade 9 Mathematics teacher, was concerned by the poor performance and the high failure rates of learners in mathematics (Naidoo, 2007) and raised questions about the effectiveness of her feedback in her mathematics classroom. She was concerned about whether feedback was communicated in a way that was useful to learners. She also wanted to find out whether certain forms of feedback information were more useful than others. Naidoo's concern about the effectiveness of feedback arose from her observations of learners who often repeated the mistakes even after receiving feedback on how their mistakes could be corrected (Naidoo, 2007). Naidoo used journals and interviews to probe the learners in her class about their opinions of assessment feedback.

She found that some answers she received were sometimes supported and at other times contradicted by findings reported in the literature. Naidoo was surprised by the depth of the learners' understanding of assessment. She found that her learners' understanding of teacher feedback "provided insights into the value and purposes of feedback" as identified by researchers and educationists (Naidoo, 2007, p.87). She found in the interviews that learners wanted guidance on how they could improve, and comments such as 'good' or 'excellent', were not seen as particularly helpful to them. The learners also spoke about the negative effect that some comments had on them. This view is supported by Young (2000, p.4) who wrote that verbal comments that are derogatory are viewed as "absolutely annihilating" (Young, 2000, p.414) for the learner in the learning experience. Other findings about the conceptions held by learners about the role of feedback in providing directions on how to proceed with a calculation, improving understanding of a mathematics concept and filling in gaps in understanding were supported by research findings from other countries.

A finding not reported in literature, was the value of written feedback because they could "read and make sense of what is written at a later stage", thereby

constituting a resource for further learning (Naidoo, 2007, p.94). There were other findings that were different from what was expected. The study identified a learner who did not welcome her teachers' unsolicited feedback, especially if she was busy concentrating on her work. She wrote that she was irritated if the teacher disturbed her concentration just to mark her work. The answers that Naidoo received were not always expected but this enabled her

to develop a personal knowledge of her learners' expectations of the teacher's role in helping them develop their mathematical understanding.

### Unpacking learners' calculations

Khan, a Grade 9 mathematics teacher was concerned that her learners often performed badly in the Common Tasks for Assessment (CTA), yet she noticed that their performance in the classroom based Continuous Assessment (CA) component was reasonable. The means of the class for the average of the CA and the CTA were 58 and 34 respectively, a 24 percentage point difference, showing that on average learners only achieved 58% of their CA marks in the CTA.

Her concern over the discrepancies in performance between the two assessments led her to investigate why her learners performed more poorly in the CTA than in the CA (Khan, 2009). She found that the task design of setting each sub-task within one extended context, was problematic for the learners. The learners struggled with resultant language load used to portray the context. The learners struggled to identify the crucial information from all the extraneous details associated with the contextualisation. The readability levels of certain instructions were beyond the average Grade 9 level (when measured using an easily available readability index).

Khan also did analyses of errors similar to that described by Ball *et al.*, (2008, pp. 397, 400) as being characteristic of the distinctive work done by mathematic teachers and which forms part of what they term specialised content knowledge. The figure below consists of a task followed by a learner's (Cleo) written response.

Figure 2: A learners' response to a contextualised task (DoE, 2005, p.6)

**Activity 1.4** Individual

Recommended time: 30 min ✓ Marks: 17

1.4.1 (a) Use the map on page 4 to estimate the distance from your school to the marked entrance to the Kruger National Park. (5)

(b) Why can the answer in (a) only be an estimation? Give two reasons. (2)

1.4.2 When the CTA was copied at a certain school the map on page 4 was reduced to 80%. What influence can this have on your answer in 1.4.1? Explain your answer. (3)

1.4.3 Up to 1994 the KNP stretched 350 km along the Mozambican border and was on average 60 km wide. Use this information to determine the approximate area of the Park before 1994, in km<sup>2</sup>. (2)

1.4.4 According to one source, the actual area of the Kruger National Park before 1994 was 2 149 700 hectares. Convert your answer from 1.4.3 to hectares and explain why your answer differs from the actual area. (2)

1.4.5 According to an agreement with the governments of Mozambique and Zimbabwe, the Kruger National Park will become part of the Greater Limpopo Transfrontier Park. The eventual size of this Park will be 100 000 km<sup>2</sup>. Calculate the percentage increase of this Park compared to the size of the Kruger National Park before 1994 (2 149 700 ha). (3)

Activity 1.4

1.4.1 a)  $40 \times 16 = 640$  - The distance from my school. 2

b) It because I draw a straight line  
 ↳ The map is also showing

1.4.2 The answer can change because the distance can also change. ✓

1.4.3 ~~the final amount in 1994 distance~~  $350 \times 60 = 21,000$

1.4.4 168100 the actual answer was 410 and I convert it.

1.4.5 21497 / 462,121009 the 3 is the increasing mark.

In trying to understand the learners' response to Question 1.4.4, Khan inferred:

For Question 1.4.4, Cleo extracted the numbers 350 and 60 (representing the length and width of the Park) from Question 1.4.3. She proceeded to add the length and width obtaining the number 410, which she then squared to obtain a value of 168100. It is evident that Cleo was struggling to make sense of question 1.4.4. The instruction "*Convert your answer from 1.4.3*" seems to have cued her to extract the numbers 350 and 60 from 1.4.3. Furthermore her comment "*the actual answer was 410 and I converted it*" reveals that she interpreted the instruction "Convert your answer" to mean, "*square the result*".

For Question 1.4.5. Cleo responded by writing 21497/462,121009. It is a fascinating exercise to trace how she arrived at these figures. Firstly, Cleo extracted the 2149700 (representing the original area in hectares) and divided it by 100 to arrive at the 21497 in the numerator. To get the number in the denominator, she extracted the only two figures of 2149700 and 10 000 (representing the original area in ha. and new area in km<sup>2</sup>), which appeared within the maze of instructions. She then divided the first number by the second and squared the result, to obtain the number 462,121009 in the denominator. This provides an indication to us, of Cleo's struggles to provide answers to questions that she did not understand (Khan and Bansilal, 2010, p.290).

Khan's error analysis was done to take her beyond seeing "answers as simply wrong" but it is an attempt to gain a "detailed mathematical understanding required for a skilful treatment of the problems students face" (Ball *et al.*, 2008, p.397). In this case the problem was the misinterpretation of the instructions and therefore the role of the language in hampering the learner's attempt at the problem. Thus Khan's (2009) study challenged her views about the use of contextualised tasks in controlled settings. When she began the study, she did not question that a national assessment protocol could be unreliable. Her experiences led to a broadening of her understanding of the need for validity, consistency and reliability of assessments, especially in a diverse context as South Africa. Her findings led her to take a critical stance, and she concluded that the CTA programme of assessment was not a fair means of assessment. She further recommended that the design should be urgently revised.

## Reasoning about the rules of differentiation

A third case is that of Pillay, a Grade 12 mathematics teacher, who was concerned over the years about her learners' lack of conceptual understanding. This concern led her to design a study to explore her Grade 12 learners' understanding of the concept of the derivative. She analysed the written responses of her class to two assessments, and interviewed four learners in a bid to understand the ways in which they responded to the tasks. By turning her attention to the notation errors made by her learners, she was able to deepen her own understanding of many of the conventions used to denote derivatives and functions. In the process of studying her learners' misuse of rules, she deepened her grasp of the relationship between the various rules of differentiation. She also observed that the presentation of a question could cue learners to produce certain responses. The learners' tendency to draw on formulae from quadratic theory, compelled her to investigate links between and within the cubic and quadratic functions. These reflections shifted and transformed her own 'big ideas' of the purpose and value of certain algorithms.

Pillay's understanding of learning theories such as mathematical proficiency (Kilpatrick, Swafford and Findell, 2001) deepened. She was able to find evidence, for example, of students' lack of procedural fluency when carrying out certain rules, while also identifying instances of learners who demonstrated conceptual understanding. She found evidence that many learners knew how to perform a procedure but did not know when to use it, such as when they used the turning point formula for finding the turning point of a quadratic function, to find the turning point of a cubic function. Many learners, when asked to find the value of the derivative of function at a certain point, found the value of the function at that point.

Her study led to a deeper understanding of links between concepts such as gradient and derivative. Her understanding of the rationale behind the sequencing and approach to certain topics was also strengthened. She was able to see the reasoning behind the introduction of calculations of the average gradient between two points on a graph a year before the introduction of the concept of the derivative.

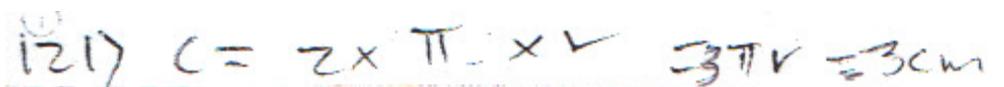
One of the assessment standards at grade eleven requires learners to investigate numerically the average gradient between two points and thereby develop an intuitive understanding of the concept of the gradient of a curve at a point. In the grade twelve year learners are required to investigate and use instantaneous rate of change. They are required to accomplish

this by first developing an intuitive understanding of the limit concept in the context of approximating the gradient of a function. This sets the stage for establishing the derivatives of various functions from first principles. With the emphasis now being on the concept of average gradient, leading to the concept of the derivative it is hoped that learners will develop a better understanding of the concept of the derivative (Pillay, 2009, p.108).

The findings of her study led her to recognise that curriculum change alone may not produce the desired conceptual understanding. Her recommendation was that curriculum change should be accompanied with a change in the type of assessment tasks that are given to learners, in order to foster conceptual understanding of the derivative. She also highlighted the role of sequencing in influencing learners' understanding. The phrases 'met-before' and 'met-after' were coined by Tall, cited in De Lima and Tall (2008), in order to describe the effect that prior learning can have on newly introduced concepts. Pillay's study revealed to her that the algorithms linked to the quadratic functions (such as calculations of the roots of a quadratic equation, turning point of a quadratic function) sometimes impacted negatively as a met-before for the algorithms linked to cubic functions. This recognition of the links between the two topics, led her to extend and revise her own big ideas of graphs of quadratic and cubic functions.

### Re-thinking the purpose and form of assessments in Mathematical Literacy

A fourth case is that of Debba (2012) whose concern about his learners' disengagement with contexts used in Mathematical Literacy (ML) assessment tasks, led him to try to identify reasons behind the disengagement. Some of his findings were unexpected and contradicted previous research showing that if students understood certain contexts they were more likely to perform better. In certain cases, learners' previous experiences of the context influenced their responses differently from what was expected from them. He also found that many of his learners were not interested in making sense of the contexts but were more interested in picking out formulae to generate answers but did not necessarily use the formula productively. For example, the excerpt below shows a learner's response to a question which asked for the circumference of a tool whose dimensions were represented in a diagram.



121)  $C = 2 \times \pi \times r$  ✓  $37.7 = 3\text{cm}$

His response when scrutinised shows that the learner did not substitute any values into the formula that was provided, but simplified the expression by ignoring the variables. He responded similarly to a second question which asked for the surface area of a figure:

B)  $SA = 2 \times \pi^2 \times r^2 + \pi \times d \times h = 3\pi r^2$   
3cm. surface area

Here too, it is clear that the learner did not substitute any value but just wrote an answer, showing again that he did not know what to do with the variables. This tendency to ignore variables in simplification of algebraic expressions is a common misconception which often manifests itself in early algebraic experiences.

Debba progressed from believing that it was unfair on learners to use contexts which were unfamiliar to them in the ML classroom, towards distinguishing between the purposes of the different contexts. His observations and reflections allowed him to distinguish between the use of contexts for developing understanding in mathematics as opposed to preparing learners to make informed decisions. His study led him to see the importance of using authentic life related contexts with learners in order to fulfil the mandate of ML, irrespective of whether they were familiar or not. This progression in the understanding of the important role played by authentic contexts changed Debba's views of ML assessments, convincing him that examinations actually detract from the fulfilment of ML aims. He argued that the learners need opportunities to engage in authentic contexts so that when they encounter these, they can make informed decisions as is the mandate of ML. However setting examination tasks around contexts which they have not encountered before, disadvantages many learners if they miss crucial information or are unable to understand certain conventions about the context. He wrote:

Hence if we want learners to engage with life related issues, we need to find alternate means of assessment rather than using examinations only. The examination setting creates pressure on learners to pass, and not to bother about making sense of what is being asked. The examination setting also restricts the kinds of authentic engagement that could help learners get to grips with real life issues (Debba, 2012, p.117).

Thus his 'big ideas' about the role of contexts in an ML classroom shifted in conjunction with his shift in understanding of the purpose and rationale behind ML.

## Discussion and concluding remarks

In this article I have presented a case of four teachers who by doing a systematic inquiry into a learning issue that they were concerned with, strengthened their own mathematics knowledge for teaching. We found that the teachers' knowledge of their learners, knowledge of the effect of sequencing, knowledge of the content, and knowledge of the curriculum were deepened.

In the case of Naidoo, by finding about her learners' understanding and preference for certain types of assessment feedback in her mathematics class (which was sometimes different from what she expected), she increased her knowledge about her learners. Her insights into the value of feedback has influenced her understanding of assessment which has allowed growth in pedagogical knowledge, which is however not a focus of the framework of Ball *et al.* (2008) but is an important knowledge area for a teacher. A significant point is that the domain of her personal knowledge of teaching has also been enlarged, by the insights she has received. Her future offerings of feedback to her learners will never be the same as it was before the study. A further point from the case of Naidoo, is that her findings were sometimes different from that reported in the literature, and her knowledge of learners originated in 'the wisdom of practice' (Shulman, 1986), and was further enhanced by the activities she engaged in as part of her research.

In the case of Khan, by engaging in the error analyses of her learners, she has enlarged her domain of *specialised content knowledge*. Khan by her in-depth analysis of learners' responses had also increased her *knowledge of content and students* by hearing and interpreting her learners' emerging thinking and their opinions and frustrations of the language overload in the tasks. Furthermore she was able to engage in critical reflections by *hunting her paradigmatic assumptions* (Brookfield, 1995) that national assessments are valid and reliable instruments that can indicate whether learners know and understand their work. She has learnt that the use of the extended context in the CTA programme, actually led to the instruments being unreliable and unfair. She has been able to take a critical stance about validity issues in assessment.

In the case of Pillay, by looking at the learner's misconceptions in using inappropriate algorithms in calculating turning points she has been able to

develop her *common content knowledge* in graphs of quadratic and cubic functions by looking at similarities and differences between the two. She has also articulated her understandings in the domain of *horizon content knowledge* by engaging in discussion of how the rules of differentiation are related to other techniques of differentiation and by seeing how the approach to the teaching of calculus has been developed in earlier years. This understanding of the linkages and sequencing of concepts in the curriculum is also part of *knowledge of content and curriculum* and can be described as vertical curriculum knowledge (Shulman, 1986, p.10).

Debba's study helped him develop an understanding of the curriculum of ML, that is he has deepened his understanding of *knowledge of content and curriculum* by re-examining his big ideas about the purpose of ML. He too, has improved his *knowledge of content and student*, because he has learnt about his learners' misconceptions as well as their approach to providing answers to the assessment tasks. Debba has also hunted his *paradigmatic assumptions* that familiarity of contexts is necessary for learners in order to help them develop ML skills, and has instead realised that teaching in the subject ML should provide opportunities for learners to engage in the unfamiliar contexts. This critical reflection has enabled him to take a critical stance on the issue of the form of assessments used in ML, and he recommends that assessment should not be done by examinations because the limited time frames mean that learners do not engage with the context, thereby actually constraining the vision of ML.

Thus in all four cases, we have seen that these teacher-researchers have deepened their mathematical knowledge for teaching in different domains, by engaging in inquiry in their own classrooms. In all these cases, a precondition for this learning was the critical reflection that they engaged in, which was facilitated by their research inquiry process. However, these teachers did not simply start to reflect on their teaching, but did so in a formal systematic way while being guided by their supervisor. Although the teachers' learning was an outcome of the research process, their detailed observations and willingness to hunt their framing assumptions enabled their learning. The teachers, as part of the research process also engaged with theory and critically assessed other research studies related to their own inquiries, and these activities would also have enhanced their learning.

The question then arises whether teachers can only learn if they are enrolled for postgraduate studies, as in the case of this sample. Are there other teachers who find that the classroom can be a powerful site for their own mathematical knowledge for teaching? Can learning occur outside of a research setting? Clearly the answer to this question is yes — like in the case of Gerty (Maoto and Wallace, 2006) where learning occurred in a research setting although she was not the researcher. A further enabling factor seems to be the presence of a concerned 'other' who is able to listen to the teachers articulate their learning while providing a non-judgemental ear. In the four cases presented in this article, the research supervisor would have played that role. In the case of Gerty it was the researcher, while in the case of Thompson and Thompson (1994; 1996), the researcher also supported the teacher in his struggles to cross the gap between his own understanding and that of his learner. A study by Peressini and Knuth (1998) described how a teacher struggled to understand a different solution method offered by a group of students. It took the intervention of a pre-service teacher to convince the teacher of the equivalence between the solutions. Thus in this case, the support to the experienced teacher was provided by a novice teacher.

In South Africa, little attention has been paid to implementing classroom teacher support. Instead there has been a focus of external interventions, which aim to 'increase' teachers' knowledge. Current discourses in South Africa seem to ignore the classroom dimension of teacher learning. Ministers, journalists, as well as Non-Governmental Organisations often speak about teachers' poor content knowledge but ask for interventions in the form of formal courses to improve the knowledge. Teachers themselves ask for more knowledge interventions, showing that they believe that they can only learn when others teach. However this article has demonstrated that the classroom can be a powerful site for teachers to improve their own knowledge. It has shown that for teachers who set out on a disciplined inquiry centred around teaching and learning issues in their classroom, one of the benefits is a deepening and shifting of their own mathematics knowledge for teaching.

Perhaps teachers themselves need to be convinced about this. There is a whole generation of teachers who have lost faith in their ability to learn by themselves and will need sustained support in order to start taking steps in their own professional growth. Bansilal and Rosenberg (2011) in their study of 41 teachers' reflective reports found little evidence of teachers reflecting on learners' learning. Many of the teachers engaged in descriptive writing and not critical reflections, suggesting that teachers will need much support in order to

encourage them to take on their own learning by observing, reflecting, sharing and learning. Research (Davis, 1997; Clark and Linder, 2006; Maoto and Wallace, 2006) emphasises that a collegial and supportive environment is crucial to teachers' growth, comfort and well-being. Thompson and Thompson (1996, p.19) have also cautioned that the level of support teachers need in order to develop such a conceptual orientation to teacher knowledge "far exceeds what most teacher enhancement and teacher education programme provides". However such a model that helps teachers

take responsibility for their own learning should be prioritised by education authorities because of the immense benefits it offers.

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# Exploring the views of educators and students on privileged knowledge domains in a teacher education programme: a case study

Lungi Sosibo

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## Abstract

The reported case study solicited the views of teacher educators and students on the teacher knowledge taught and the way it was imparted in a teacher education programme that offered commercial subjects at the Bachelor of Education degree level. The objective was to establish the extent to which the knowledge domains were interconnected or whether some were prioritised. Data was collected from twenty students and seven educators using semi-structured interviews, and analysed using deductive qualitative analysis method. The Minimum Requirements for Teacher Education Qualifications policy framework was used as a source of codes and the five domains of the framework were used as *a priori* codes under which analysed data was placed. The findings confirmed that some knowledge domains were prioritised. A disjuncture was discerned between teacher knowledge and practical experiences of students from disadvantaged backgrounds. The study concludes with a discussion which has implications for teacher education programmes.

## Introduction

Different conceptions of teacher knowledge abound. Verloop, Van Driel and Meijer (2001, p.443) provide an all-encompassing definition of teacher knowledge base as “all profession-related insights that are potentially relevant to the teacher’s activities”. Grossman and Richert (1988, p.54, in Ben-Peretz, 2011, p.8) extend this definition by breaking it down to “knowledge of general pedagogical principles and skills and knowledge of the subject matter to be taught”. In other definitions, teacher knowledge is classified into elaborate categories that include a variety of domains such as subject matter/content, curriculum, pedagogy, contexts, learners, formal and material elements of teaching, etc. (see, for instance, Feiman-Nemser, 2001; Cochran-Smith and Lytle, 1999; Shulman, 1987).

In South Africa, the Department of Higher Education and Training (DHET) (2011) has developed a policy framework on the Minimum Requirement for Teacher Education Qualifications (MRTEQ). This framework serves as a blueprint for the knowledge that every prospective teacher has to acquire before they become certified professionally. The framework stipulates five teacher domains, namely, disciplinary, general pedagogical, practical, fundamental and situational knowledge. These will be described in more depth in the literature review. Of the five, the fundamental and situational domains could be viewed as an attempt to redress the imbalances of the past that were created by the apartheid system. During the apartheid era, indigenous languages were denigrated and only English and Afrikaans were given priority and status in the South African education system. Similarly, contexts played a significant role in dividing the South African education system socio-economically into haves and have-nots.

Currently, not much has changed as the quintile system still divides South African public schools into fee-paying and non-fee-paying, depending on the socio-economic status of the community in which the school is situated. In such school contexts, Information and Communication Technology (ICT) creates a great digital divide, with impoverished schools struggling to acquire and have access to technology while affluent schools are well-equipped (Gudmundsdottir, 2010; Prince, 2007; Department of Education, 2003).

As the MRTEQ policy framework was gazetted only in 2011, research which investigates whether the domains as stipulated therein are currently taught in teacher education programmes (TEPs) is scarce. Yet such research may provide information on whether the thinking by teacher educators and policymakers is similar or not regarding teacher knowledge, itself so vital for teacher education. The purpose of this study was to investigate teacher knowledge domains that were taught in a TEP. In particular, this research sought answers to these questions: (1) What domains of teacher knowledge are offered in the teacher education programme? (2) How are they taught? (3) To what extent are they interconnected? (4) Are there knowledge domains that are prioritised?

South African teacher education is still in the process of redefining itself in the midst of new policy frameworks, as well as benchmarking itself against the standards set out by the international world. Against this background, research such as undertaken in the current study is significant as it sheds light on how teacher knowledge is handled in one TEP.

This paper is organised into five parts. The first part presents the literature review, followed by research design and methodology, and findings. Lastly, the discussion is presented and conclusions are drawn.

## Literature review

This section discusses pertinent literature on teacher knowledge and includes the views of Shulman (1987); Feiman-Nemser (2001); Cochran-Smith and Lytle (1999); Morrow (2007a,b) and the MRTEQ policy framework (DHET, 2011). Shulman classifies teacher knowledge into seven domains.

Emphasising the importance of a deeper understanding of content knowledge or subject matter knowledge (SMK), Shulman (p.xiii) states that teachers must “be well educated, especially in the subject matter content they teach, and their career-long professional educational experiences must continue to be grounded in the centrality of that content”. He further identifies general pedagogical knowledge (GPK) which is a combination of classroom management and organisational principles and strategies. Curriculum knowledge (CK) involves an understanding of materials and programs that, according to Shulman, serve as ‘tools of the trade’ for teachers. Pedagogical content knowledge (PCK) combines content and pedagogical domains.

Shulman also proposes knowledge of learners and their cultures, characteristics and needs, which is crucial as learners’ backgrounds are heterogeneous. Gorski (2009) refers to this domain as multiculturalism knowledge. Knowledge of varying educational contexts is equally significant as learners’ schooling backgrounds are considerably different. The challenge is for teachers to find ways of adapting their teaching approaches to suit learners who come from and learn in these varying contexts. The last domain is knowledge of educational ends, purposes, and values, and their philosophical and historical grounds.

Unlike Shulman whose focus is on the classification of teacher knowledge, Feiman-Nemser (2001) is more concerned with the developmental aspects of these domains in students. She thus proposes ‘central tasks’ aimed at developing teacher knowledge at each stage in the continuum of teacher preparation learning. This is important as existing research reveals that neophytes deal with anxiety and developmental concerns at each stage of teacher preparation.

According to Feiman-Nemser, the central tasks of learning to teach should include the development of:

- i. Knowledge which allows pre-service teachers to examine and critically analyse their existing and taken-for-granted beliefs and assumptions about teaching and learning, students and content so that they could unlearn existing conceptions and develop new visions of good teaching;
- ii. Subject-matter knowledge which she divides into three domains:
  - (i) “central facts, concepts, theories, and procedures within a given field;
  - (ii) explanatory frameworks that organize and connect ideas; (iii) rules of evidence and proof” (p.1017).
- iii. An understanding of learners, human development and diversity which helps the neophytes to question their own taken-for-granted assumptions and stereotypes about their learners so that they could adjust teaching and learning accordingly;
- iv. A beginning repertoire which enables students to master issues related to curriculum; approaches to effective teaching, learning and assessment; and to make sound judgements;
- v. The tools and disposition to study teaching, for which Feiman-Nemser suggests the skills of observation, interpretation and analysis. Evidently, lack of these skills results in teachers who can neither think nor reflect critically.

Although her focus is on the development of teacher knowledge, it is obvious that Feiman-Nemser’s domains (ii), (iii) and (iv) correspond closely with Shulman’s SMK, CK and GPK classifications. What sets her domains (i) and (v) apart from Shulman’s are the inherent developmental aspects in them.

Cochran-Smith and Lytle (1999) distinguish between knowledge-for-practice, knowledge-in-practice and knowledge-of-practice. Knowledge-for-practice assumes that there is a unique teacher knowledge that distinguishes professional educators from laypersons or people practising other professions. This view is not far removed from Shulman’s and Feiman-Nemser’s. Fenstermacher (1994, in Verloop *et al.*, 2001) calls this domain knowledge *for* teachers as it is prescribed for teachers. Knowledge-for-practice is traditional

as it portrays teachers as consumers of knowledge that has been generated for them by mostly university-based researchers and scholars. This view justifies the preponderance of teaching guides and handbooks that are used in teacher-preparation and teacher-professional development programmes.

Knowledge-in-practice conceives of teacher knowledge as grounded in the practice of teaching and learning and as such, cannot be divorced from that practice. Cochran-Smith and Lytle conceive of teachers as acquiring teaching knowledge and skills from the profession itself “through experience and through considered and deliberative reflection about or inquiry into experience” (p.262). This view of knowledge is similar to Schön’s (1983) perceptions of teachers as practitioners who reflect in-, from- and on-action.

On the other hand, knowledge-of-practice or knowledge *of* teachers (Fenstermacher, 1994, in Verloop *et al.*, 2001) assumes that teachers are central in generating their personal teacher knowledge. This domain is consistent with the principles of constructivism and acknowledges teachers as critical and reflective actors, researchers, investigators and creators of knowledge they use in the classrooms. Referring to this domain, Cochran-Smith and Lytle (1999, p.272) contend that “knowledge making is understood as a pedagogic act- constructed in the context of use, [and] intimately connected to the knower”.

Morrow (2007a,b) distinguishes between formal and material elements of teaching, the former which he defines as non-context bound and the latter as context bound. Morrow (2007a) conceives of teacher education as preparing teachers for the professional functions of organising learning systematically so that learners can order, understand and grasp the information they learn. For this to be possible, Morrow (*ibid*, p.72) suggests that teachers should possess professional competences such as programme design and assessment strategies, including providing constructive feedback to learners. In his view, the function of teacher education is to enable and nourish teachers to accomplish this task. Morrow (*ibid*, p.100), however, contends that in our teacher education programmes “we repeatedly define the work of teachers in terms of its material elements” which involve contexts, facilities, resources, conditions, etc. By so doing, we prepare teachers to teach specific learning areas in a specific phase using certain approaches, thus limiting them to specific contexts and precluding them from functioning effectively in any teaching and learning environments.

The Policy on the MRTEQ (DHET, 2011) stipulates the knowledge base for pre-service teachers which resembles Shulman's categorisation and includes:

- i. Disciplinary knowledge or subject matter knowledge, which includes education and its foundations;
- ii. General pedagogical knowledge which involves knowledge of learners, teaching, learning, curriculum, assessment strategies, as well as specialised pedagogical content knowledge which includes methods, strategies, rules and principles of a discipline;
- iii. Practical knowledge or "learning in and from practice" (p.8);
- iv. Fundamental knowledge or knowledge of languages, information communication technologies (ICTs) and academic literacies;
- v. Situational knowledge or knowledge about contexts, situations, settings or environments for the purpose of meeting the diverse needs of learners.

The review of literature presented different perspectives on what counts as teacher knowledge base. The dominant view is that there exists a body of teacher knowledge out there that makes teachers or the teaching profession unique from other professions. Be that as it may, there seems to be another perception of teacher knowledge as developing the neophytes' critical and reflective practice skills. Furthermore, there is a view of teacher knowledge as tools of the trade that facilitate the process of becoming a teacher. A further perception is that of material elements of teaching as confining teachers in certain specific conditions. What is not clear in the literature is whether all teacher knowledge is or should be acquired in university classrooms or whether it can be learnt on-the-job. If it is the latter, how can it be taught and learnt systematically by all students in such environments?

The MRTEQ framework guided this research because in the near future it will undergird all TEPs in the South African higher education and training (HET) landscape. Using this framework will help to determine how far the domains of teaching offered in one TEP are from the norm as required by the policy (MRTEQ). This will ascertain the amount of work that needs to be done in reconceptualising and redesigning a new curriculum for this TEP in accordance with the stipulations of this policy framework.

## Research design and methodology

### Design and sampling

The design was a case study of a single TEP. Data was collected from a purposive sample of twenty students selected from first, second, third and fourth years, with five students from each of the TEP levels and seven educators who taught in the TEP. Because of the small number of academic staff in this TEP, the sample size of seven out of nine educators was justified. Factors of race, culture and gender were considered in the selection of students in order to provide diversity as it was believed that the knowledge being taught to them might have impacted on their growth and development in various ways.

This study was undertaken in the Department of Further Education and Training (FET) TEP which offers commercial subjects at the level of a Bachelor of Education (B.Ed.) degree. The TEP is located within a Faculty of Education (FoE) at a higher education institution (HEI) situated in the Western Province. The subject offerings in the TEP are classified according to knowledge domains that pre-service teachers are expected to master before they become credentialed (Appendix 1). Appendix 1 is self-explanatory. It illustrates teacher knowledge offerings at each level as well as subject choices that students have to make. As can be seen, some teacher knowledge domains are compulsory and others are optional.

Educators of subject matter knowledge (SMK) also teach subject didactics or specific pedagogical knowledge (SPK) related to that subject. For example, mathematics educators teach mathematics specific pedagogical knowledge. Thus, students learn specific teaching skills (or PCK) relevant to their majors in SPK. In general pedagogical knowledge (GPK) students learn general teaching skills such as classroom management, and teaching and learning theories and strategies. Practical knowledge (PK) or professional studies mainly prepares students for practice teaching (TP). They mainly learn to develop TP portfolios. Students are placed in schools for TP for a period ranging from four weeks to six months.

## Data collection

Views regarding the knowledge domains taught and how they were taught were collected using in-depth, semi-structured interviews. Boyce and Neale (2006) contend that in-depth semi-structured interviews are useful when you want detailed information about a person's thoughts and behaviour or to explore new issues in depth. During the interviews, probing was done in order to clarify any ambiguities. A sample of interview questions appears in Appendix 2. Each interview lasted for a period of approximately 60 minutes. Interviews were tape-recorded with the permission of the participants and transcribed *verbatim*.

## Data coding and analysis

Data was analysed using deductive qualitative analysis (DQA) (Gilgun, 2011; Acock, Van Dulmen, Allen and Piercy, 2005 in Bengson, Anderson, Allen, Acock and Klein, 2005). In DQA, researchers use a theory or theoretical framework to guide their research. The MRTEQ framework guided this research and became the source of codes that I used to analyse data. The five domains of the MRTEQ framework were *a priori* codes under which analysed data was placed. Axial coding helped me to identify common emergent themes. These themes were colour-coded, categorised, grouped together and placed under MRTEQ codes. Gilgun describes negative case analysis (NCA) as a procedure which helps researchers to look for data that does not fit with the theory. During data collection and analysis I conducted NCA to check for data that did not fit with the MRTEQ codes. No such data was found. Using DQA helped me to focus my research question and to benchmark the knowledge domains against the MRTEQ framework. Member checking was used to enhance the validity and credibility of analysis.

## Ethical issues

Ethical clearance was obtained from the Ethics Committee of the Faculty of Education from which the research participants were drawn. Participants were informed about the confidentiality of information gathered and the voluntary nature of their participation. Throughout the study, pseudonyms were used in order to protect the participants' identities.

## Findings

Emergent themes which were categorised under five teacher knowledge domains of the MRTEQ framework are discussed in the findings. They include these domains: disciplinary; fundamental; practical; situational and knowledge about learners; as well as pedagogy.

### Disciplinary knowledge

Interviews with educators revealed that disciplinary knowledge was supported in the TEP, as shown by the six major subjects in Appendix 1. Evidently, educators paid particular attention to the extent to which content knowledge taught at university was relevant to that in schools. They mentioned that their collaborations and networks with the Western Cape Department of Education (WCED) helped them to accomplish this. They hinted that they discussed curriculum-related issues and policy frameworks in the Advisory Committee meetings where WCED subject advisors and teacher educators met. These educators perceived collaborations as instrumental in strengthening the knowledge base imparted to the students. Marie, the accounting educator described the importance of this collaboration as follows:

I work closely with the accounting subject advisor to ensure that I stay abreast of the content which is required of accounting teachers to teach in a particular academic year. The objective of the aforementioned is to ensure that our students are prepared well to teach the topics during teaching practice according to the pacesetter for that specific academic year.

Jacques, the CAT educator who claimed that he regularly attended Advisory **Committee meetings and provided in-service training to high school teachers**, reiterated the importance of these collaborations. He conceptualised the role of subject advisors as critical in keeping teacher educators abreast of disciplinary and theoretical developments in their respective subjects. In his view, teachers are mentors who provide emotional support and conceptual knowledge to the students. He described the motive for his involvement in these collaborations as key in helping him ascertain if the content he taught was relevant or not.

Educators mentioned that the subject advisors perused their subject guides to ascertain whether disciplinary knowledge imparted in TEPs was compatible with that in schools. Bennie confirmed and resoundingly agreed that:

By involving subject advisors, we are sure that the content we teach corresponds with that taught in schools. By working together, we avoid a situation which could potentially frustrate students, learners, mentors and the DoE, especially if the content is different from that taught in schools.

The importance of matching disciplinary teacher knowledge with school-based knowledge was reiterated by Marie, the accounting educator who explained that:

Students studying accounting levels 1 and 2 focus on the content which they are required to teach from Grade 10 to 12. In the third year they focus on content that delves deeper into the aspects of accounting which are not necessarily required at school level.

John, a mathematics educator mentioned that teacher knowledge taught in this subject transcended that taught in school and that it continuously aimed at clarifying learner misconceptions. In his view, mathematics teachers should teach the content in a way that learners would understand rather than present problematic algebraic and geometrical topics in a rote fashion, as often happens in school. He further suggested that mathematics teachers should sequence mathematics activities, taking into consideration the learners' prior knowledge. Additionally, he argued that many mathematics teachers do not know how to assess this subject and recommended the use of Bloom's taxonomy.

Students highlighted the emphasis that educators placed on teacher knowledge domains in their TEP. Nonetheless, some of them expressed dissatisfaction with the fact that their major subject content ended at third year of study whereas specific and general pedagogical knowledge (SPK and GPK) continued up to final year. The concern they raised was that they would not have enough content knowledge when they graduate. Perhaps this explains the imbalance in the emphasis placed on different knowledge domains in the TEP, and the fact that pedagogical knowledge is prioritised over content knowledge. None of the educators mentioned this discrepancy although the accounting educator had insinuated it by being silent about what happened to accounting content knowledge at fourth year level. However, since no direct question had been asked about why content was offered only up to third year, it might be possible that the educators did not perceive it as a source of concern to the students.

## Fundamental knowledge

With regard to fundamental knowledge, educators did not give much input. However, students expressed sentiments that pointed to the fact that this knowledge was not prioritised in the TEP. They felt that educators paid lip service to the fundamental knowledge as languages and information communication technologies (ICTs) or computer literacy were offered as non-major subjects and did not incorporate pedagogical knowledge. This implies that students cannot teach these subjects as they do other commercial subjects; as languages are taught solely for communication and learning and teaching (LoLT) purposes, and ICTs only for skills development. Students further raised concern that these subjects were taught only up to second and third year levels, respectively, which they said meant they did not acquire adequate skills in them. It is possible that the programme planners do not perceive fundamental knowledge as central to the development of future teachers. Alternatively, students might be regarding them as more important than mere fundamentals.

## Practical knowledge

This domain seemed to receive the highest priority, judging by the efforts made by educators to improve it. These efforts included the use of a debriefing form and initiatives such as TP funded projects, and the use of micro lessons, as discussed below.

Several educators mentioned the debriefing forms which they said were developed in order to provide students with coaching or scaffolding after they had presented TP lessons. This form consisted of two sections. The first required students to reflect on their performance by giving input on the lesson aspects in which, according to their judgment and based on the criteria on the debriefing form, they had done well or not well during lesson presentations. The second required educators to give students feedback on the same items. The reflection was guided by a conceptual framework or criteria against which a student's performance was benchmarked.

One educator explained that this form was developed in reaction to students' dissatisfaction with evaluators who allegedly failed to provide them with detailed feedback after TP evaluations. Educators also indicated that the

instrument was used to facilitate students' reflection on their performance in order to enhance their professional growth and development as reflective practitioners.

Educators had different perceptions about the debriefing forms as developmental tools. According to Bennie, a GPK and professional studies (practical knowledge) educator, the debriefing forms facilitated interactions between students and TP evaluators which he said was vital for students' professional growth as future teachers. This view resonates with that of Brady, Segal, Bamford and Deer (1998) who perceive dynamic interaction between tertiary advisor and student facilitating growth. Another teacher educator, Jacques, described the debriefing form as,

An analytical tool that affords students with an opportunity to reflect theoretically upon their teaching performance in terms of handling content, methods, theory and practice and learner differentiation, with the sole purpose of enhancing their personal, academic and professional development.

Riana, the GPK educator who initiated the debriefing form had this to say:

Its basic tenet is to allow the students to practice reflection on their overall TP performance in terms of handling the teaching and learning process. It helps them to think about what they do, how they do it and why they do it the way they do. The form basically teaches them the main ingredients of teaching and learning.

The perspectives of students on this instrument were not much different, as shown by Jabu's utterances below:

The debriefing form makes both lecturers and us accountable for our learning. We learn to answer for what we taught, how we taught it and why we taught it that way. It also makes the lecturers take care of our learning and not leave us in the dark. I think it should be made compulsory in all teacher education programmes.

Another effort included the TP funded projects. Educators make videos of demonstration lessons which they said they converted into DVDs and web-based materials for use during professional studies periods or at students' leisure. Educators expressed a belief that students learn practical teaching skills such as reflection, handling of content, dealing with learners and pedagogical skills from these materials. Students also articulated satisfaction and admitted that the reflective reports they wrote based on the lessons added new knowledge to their TP repertoire.

The TP award is a new initiative that aims to promote students' practical knowledge. Riana reported that a student who presents a lesson that demonstrates exemplary TP based on a set of criteria such as evidence of disciplinary, pedagogical, fundamental and situational knowledge, as included in the MRTEQ, receives this award. The product of this initiative is DVDs which students, especially those at lower levels of the TEP unanimously agreed provided them with skills that promoted their professional development.

The other initiative is intended to provide assistance and guidance to the struggling students who are at risk of failing TP due to inadequate practical knowledge and skills. Students who need this assistance are either identified by TP evaluators or volunteer themselves. Before TP evaluations take place, they receive guidance and assistance on developing lesson plans and teaching techniques from fellow students whom the evaluators identify as excelling in TP. Interviews with some of these students revealed that the intervention enhanced their practical knowledge and boosted their confidence tremendously.

Micro lessons are perceived by educators and students alike as one of the best strategies for enhancing the students' practical knowledge base. Describing her gains from micro lessons, one of the students, Zama, stated:

Micro lessons help me to practise and internalise the skills and content I have learnt in class. I learn to sort, sequence and organise the content. The critical feedback I receive or give to my peers helps me to grow intellectually as a teacher.

This view supports the fact that strategies such as micro lessons should be encouraged in the TEP.

### Situational knowledge and knowledge about learners

With regard to situational knowledge and knowledge about learners, very little was mentioned by educators. This might point to the low priority given to this knowledge in the TEP. This view is supported by the fact that this knowledge is excluded from the domains that are offered in this TEP. In contrast, some Black and Coloured students gave tremendous input on this domain.

To be precise, only two educators, John the mathematics educator and Sarah the foundations educator highlighted this domain. According to John, it was critical to address classroom realities, such as poor content knowledge of teachers in impoverished classrooms and rote learning or ‘rules without reason’, which he claimed are prevalent in such contexts. However, he declared that he did not know how to address classroom diversity in mathematics teaching. Sarah reported that in her foundations of education class she taught these domains by giving her students assignments that required them to scaffold slow learners as she realised that learners do not learn the same way. She noted that differentiation should occur and that teachers should understand how to handle content when teaching slow learners as those may come from disadvantaged backgrounds.

Some Black and Coloured students articulated situational barriers or impediments that they believed impacted on their acquisition of teacher knowledge. At the heart of their concerns were philosophical and cultural ideologies that they believed were embedded in some teacher knowledge, and failure of educators to contextualise teacher knowledge to suit their backgrounds. The following utterance made by Sello expressed his concern regarding cultural impediments:

What our lecturers teach us shows that they don’t understand us, our backgrounds and our school contexts. For instance, some of the theories taught to us are too American and fail to reflect our African identities and those of the African learners. As a result, African student teachers are subjected to a ‘sink or swim’ approach as they find that some of these theories were not meant to be used within their cultures. For example, a theory that stresses the *laissez faire* approach contradicts our strict disciplinary approach in raising the African child.

Students believed that knowledge about contexts was not emphasised in university classrooms. They mentioned that this knowledge was very important since they were placed in schools with varying contexts. During the interviews, Marlin suggested that classroom context was key to pre-service teachers’ growth and development. He proposed that this knowledge should be contextualised rather than treated ‘one-size-fits-all’ as, in his view, there was a vast difference between how one taught in better versus poorly-equipped schools. He contested as follows:

I have to stress this though that at university what we are taught and the way it’s taught is as if we are all going to teach in better equipped schools, neglecting the sad reality that some of us have to teach where there is not even enough learning facilities for learners. Surely, the way one has to teach in these schools can’t be the same even though the content is the same.

Similar sentiments were shared by Kayla who asserted that teacher knowledge was not in sync with the realities in impoverished schools as it prepared them only for ‘functional’ schools. She contended that the ability to apply teacher knowledge was inextricably linked to availability or lack of resources. While calling for students to be placed only in ‘functional’ schools where they would be able to apply this knowledge, she questioned whether such placements would reflect and represent South African school realities.

Students also identified TP placements as an example of how situational knowledge was ignored in their TEP. In this TEP, students select schools in which they prefer to be placed. More often than not they choose schools that are similar to their own backgrounds. Consequently, they miss exposure to diversity. It was precisely for this reason that Damon, a Coloured student expressed this sentiment:

It’s as if White students in this new South Africa are going to remain in white schools and Black students are going to remain in black schools. What will happen if the only available job for me was in an impoverished black township school? I won’t have the vaguest idea of what to do because I’ve never been placed there and I’ve never been taught to function effectively there.

Clearly, this student’s concern raises issues that call for diverse placements that may allow students to learn to function in varying contexts. It is also worth noting that the students’ focus was more on dealing with asymmetrical school contexts than with learners in these environments. In reality, their statements might pave a way for a more inclusive teacher knowledge curriculum that focuses on situational knowledge. As shown in Appendix 1, the TEP does not offer this knowledge base.

## Pedagogical knowledge

Although specific, general and curriculum knowledge domains are not similar, educators tended to discuss them simultaneously. It was apparent that some educators expected educators of specific pedagogical knowledge to cover general pedagogical, practical and foundational knowledge domains as well. Yet in the TEP offerings, these domains, except curriculum knowledge which is not included, are clearly demarcated. Be that as it may, it is easy to make this assumption as these domains all do, to a certain extent, fall under the realm of pedagogy or pedagogical knowledge which includes teaching and learning approaches and strategies, classroom organisation and management,

curriculum development, assessment and any other skills that help teachers organise learning systematically.

According to the educators, one of the ways in which they promoted pedagogical knowledge was through active participation in the workshops where discussions of the DHET's national curriculum policy frameworks such as MRTEQ and Department of Basic Education and Training's (DBET) Curriculum and Assessment Policy Assessment (CAPS) took place. They hinted that these workshops occurred at national, regional, inter-varsity, institutional, faculty and departmental levels. Pieter, the economics educator reiterated the significance of their participation in these discourses:

The DoE frameworks serve as a theory which guides the content and methods of imparting knowledge to our future teachers. Any deviations from these frameworks might have serious implications for the information we impart to our students, how they receive it, as well as how they interpret and transfer it to the schools. This explains why it is crucial for all of us to understand the contents of these frameworks so that we could transfer them to our students.

Other educators cited classroom activities as viable strategies of imparting specific pedagogical knowledge among students. Marie, the accounting educator explained the way in which she handled specific pedagogical knowledge in her subject and how students applied it:

In my accounting didactics class, I mainly focus on one particular teaching methodology in a particular academic year. Year 1: Direct instruction, Year 2: Simulation, Year 3: Inquiry and year 4: Co-operative. Each student gets an opportunity to apply this methodology by introducing a topic required to teach at school level. In didactics first year I focus on Grade 8 and 9 EMS, second year the Grade 10 accounting syllabus, third year Grade 11 and fourth year Grade 12.

The mathematics educator emphasised the importance of teaching specific pedagogical knowledge to his mathematics students. He cited different concepts related to the pedagogy of his subject which he said were essential in mathematics teaching. These included learner motivation and enjoyment; understanding of conceptual and procedural knowledge; modelling; discussion and reflection; relationship between practising of newly learnt mathematics content and progress; and lesson planning, sequencing, integration and assessment. Surprisingly, he admitted that he did not include theories in mathematics pedagogy as he believed these were taught in other related domains such as general pedagogy, foundations and professional

studies. Instead, he indicated that he made theories practical without naming them, something he called 'Theory in practice' or 'Making theory practical'.

## Discussion and conclusions

This study investigated the knowledge base taught in a TEP, how it was taught with a view to establishing whether it was taught coherently or whether some knowledge domains were prioritised. Data from interviews was categorised under the domains of the MRTEQ framework.

Findings confirmed that some knowledge domains were prioritised and others were compromised. For instance, the importance of situational knowledge runs through the thread of the literature conducted for this study. However, it receives low priority in this TEP, judging by the preponderance of students' concerns about the absence of this knowledge in the curriculum and by educators' silence on this domain. Morrow (2007a) argues that teacher knowledge should be responsive to the changing needs. As can be seen in the students' statements, this TEP needs to respond to the current conditions and expressed needs by prioritising situational knowledge. Of further and serious concern is the low priority given to ICTs and LoLT as these are offered only up to second year and L2 up to third year, respectively, and no pedagogical knowledge related to them is offered. As shown earlier, even though these knowledge domains fall under fundamentals, students' comments seemed to suggest that they view them in a more serious light than sheer basic knowledge. Granted, no TEP can cover all the knowledge domains but the curriculum needs to be altered as circumstances demand it.

South African classrooms are inclusive and need teachers who are equipped with skills of dealing with different contexts and learners. Thus, inclusion of this knowledge base is inevitable in the context of South African teacher education, as can be deduced from the specifications of the MRTEQ which is a blueprint for TEPs in this country. Unfortunately, it would be too risky to assume that students would acquire this knowledge and skills of applying it on the job as mentors and teachers may themselves not possess these skills to model them to the neophytes. As a bandage solution, life orientation and inclusive education teacher educators could incorporate situational knowledge in their curriculum. More importantly, however, is that curriculum developers

realise the extent of biases inherent in the knowledge base offered in this TEP.

Data analysis revealed a preponderance of support for practical teacher knowledge, judging by the number of initiatives educators engage in to promote this knowledge. This domain, together with general pedagogical and specific pedagogical knowledge domains are taught up to final year. As was described earlier, these domains provide students with know-how of teaching. Yet subject matter knowledge is taught up to third year of study. This situation could reflect that the theoretical knowledge is seen as surpassing the practice that draws on it. Morrow (2007a) has this to say about this practice “there is a fashion of emphasising ‘process’ to the detriment of ‘content’...process without content is vacuous” (p.66).

Morrow (2007a, p.82) further contends that,

In Teacher Education there is a strong tendency to pay insufficient attention to *what* is to be taught, to construe teaching and learning as generic activities, with scant reference to the *content* of what is being taught or, learned. In our situation, an underemphasis on the content of teaching is a prevalent and serious problem in schools and other institutions... In the case of teaching, the teacher must know the content being taught. Content knowledge is a precondition for any teaching.

Students might have held the same view, judging by their utterances regarding content subjects ending at third year of study. They might have seen the danger of not being able to master it within such limited period of time. Without adequate content skills, they are likely not to “enable [learners] access to it” (Morrow, 2007a, p.82). In Hayes, Capel and Katene’s study (2008), student teachers and mentors also perceived content knowledge as being the most important of all teacher knowledge.

The other question is whether teacher knowledge taught in the TEP is interconnected. Feiman-Nemser mentions the connective tissue which she claims is missing in teacher education. Findings in this study did not confirm the presence of a connecting thread or a set of organising concepts/framework in the TEP offerings. The only thread that seemed to bind the TEP into a coherent entity was the strong and consistent focus on practical knowledge by the majority of teacher educators. From the interviews it was apparent that each educator does her own thing without knowing what and how others do it. Only John seemed to suggest that interconnections might exist in this TEP’s knowledge offerings.

Granted, a TEP cannot cover all the domains discussed in the literature of this study. Thus, it is unlikely for South African students to grasp everything within the four-year teacher preparation period. This situation points to the significance of professional development programmes for when the students enter teaching. Unlike the current programmes, teachers need well-structured

and well-organised programmes to facilitate knowledge-in-practice (Cochran-Smith and Lytle, 1999).

Should teacher education provide students with only school-based knowledge? Only two educators concurred that the content they teach transcends school-based content. Morrow (2007a,b) highlights the problem of teacher education's emphasis on material elements of teaching, which he says emphasises conditions, circumstances, contexts and resources that confine teachers to specific subjects, phases and teaching methods. The same applies in this TEP. Students are confined in specific subjects and phases. By doing this, educators limit and bind students to specific contexts rather than prepare them to function under any conditions. As educators, we should provide students with non-context bound formal elements of teaching such as skills of designing a curriculum, planning and presenting a lesson, managing classrooms, etc. as these will enable them to organise learning systematically in any learning situation.

The limitation of this study is the limited sample size drawn from a small TEP. Consequently, the results cannot be generalised to other TEPs. Future studies should use a bigger sample to investigate the same variables. Nevertheless, the study revealed aspects that need to be given more priority in TEPs.

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**Appendix 1: Teacher knowledge domain offerings in the TEP under study**

| Knowledge domain  | Study Level                     |  |   |   |
|---|---------------------------------|--|---|---|
|   | 1                               | 2  | 3   | 4   |
| <b>Subject Matter Knowledge</b> <ul style="list-style-type: none"> <li>• Mathematics</li> <li>• Mathematical literacy</li> <li>• Economics</li> <li>• Accounting</li> <li>• Business studies</li> <li>• Computer applications technology (CAT)</li> </ul> | Select 3 majors                 | Continue with 2 from 1st year                | Continue with 2 from 2nd year                         | -----   |
| <b>Specific Pedagogical</b> <ul style="list-style-type: none"> <li>• EMS (for Accounting, Economics and Business Studies)</li> <li>• CAT</li> <li>• Mathematics</li> <li>• Mathematical literacy</li> </ul>   | Select 3 matching majors chosen | C<br><br>Take 2 matching your major subjects | C<br><br>Continue with 2 matching your major subjects | C<br><br>Continue with 2 matching your major subjects |
| <b>Fundamental knowledge</b> <ul style="list-style-type: none"> <li>• Computer literacy</li> <li>• Language of teaching and learning (LoLT)</li> <li>• Communication Language – L2</li> </ul>   | C                               | C  | Take communication language – L2 only                 | -----   |
| <b>General Pedagogical Knowledge</b>  | C                               | C  | C   | C   |
| Foundational Knowledge  | C                               | C  | C   | C   |
| Practical Knowledge   | C                               | C  | C   | C   |

**Key:** C – Compulsory

## **Appendix 2: Interview Questions**

### Sample questions for educators

1. What teacher knowledge domain/s do you teach?
2. What approach/strategies do you generally employ in teaching it/them?
3. Which knowledge domains does your TEP support? Why do you think so?
4. Which knowledge domains does your TEP not support? Why do you think so?
5. Which knowledge domain/s do you feel should be taught in your TEP but it's not taught?
6. Why do you think it should be taught?
7. To which domain/s do you pay the most attention in your subject? Why?
8. To which domain do you pay the least attention? Why?
9. Do you perceive any connections among the different knowledge domains?
10. Do you connect the domain you teach with other domains? If yes, how?
11. Why is it important for educators to make these connections?

### Sample questions for students

1. What knowledge domains are taught in your program?
2. Which domains do you think your program supports?
3. Do you think the focus of educators on the knowledge domains is balanced?
4. If not, on which domains do your lecturers focus more?
5. On which domains do your lecturers focus less?
6. Why do you think is the reason for the situation in 3?
7. On which knowledge domains would you like your educators to focus the most? Why?
8. Do your educators make connections among the different knowledge domains? If yes, how?
9. Why do you think it's important for educators to make these connections?



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# Ready to teach? Reflections on a South African mathematics teacher education programme

Iben Maj Christiansen

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## Abstract

In this paper, I interrogate the extent to which a current mathematics teacher education programme at University of KwaZulu-Natal prepares teachers to teach well in the regional context. In order to determine which aspects to consider in the analysis, I draw on studies of factors correlated to learner achievement in South African primary schools. First, this suggests that the consideration of context should play a strong part in our teacher education. Second, it indicates that teacher actions most strongly linked to learning – deep representations, feedback guiding learning and challenging learners on their level – only occur occasionally in KwaZulu-Natal schools, and with limited opportunity to develop mathematical proficiency. The question I raise is to what extent we prepare teachers to teach in this way, and with awareness of the context. Third, I briefly consider what other, perhaps overlooked, competencies our teachers need.

In the light of Bernstein's recognition of the centrality of evaluation in the pedagogic device, I have analysed the exam papers in the programme. My analysis utilises a pragmatically compiled bag of tools. First, I distinguish between the knowledge categories in our programme: contextual knowledge, curriculum knowledge, content knowledge, pedagogic content knowledge, and general pedagogical knowledge. Next, I explore the extent to which specialised knowledge is foregrounded in our programme, drawing on Maton's distinction between a knowledge and a knower legitimisation code. Third, by distinguishing the semantic gravity of the course content, I aim to identify how theoretical or decontextualised knowledge is linked to the practice of mathematics teaching. This enables me to consider the extent to which the programmes favour cumulative or segmented learning.

My findings indicate that the programme is strongly founded in a knowledge code, and that it covers all of the five aforementioned knowledge domains, but it needs further exploration how well these are linked within and across courses, thus providing cumulative learning. Teaching for deep representations is strongly reflected in the exam papers, both in the content knowledge and the pedagogical content knowledge components, but there is virtually no indication that providing appropriate challenges to learners is important. While students are tested on their recognition and realisation of assessing learners' level of understanding, this is not utilised in teaching students to provide appropriate feedback, nor is it used to inform the design of activities which can cater for a classroom with learners of

mixed ability or varying levels of current understanding. Furthermore, there is no assessment of the teachers' preparedness to teach for adaptive reasoning. In that respect, the programme appears not to prepare the students adequately for quality teaching. I discuss whether this knowledge mix and what is *not* taught can be seen as having an implicit student in mind, thus limiting access to relevant teacher competencies for some students.

## Introduction

One assumption behind this paper is that teaching is facilitation and organisation of systematic learning (Morrow, 1999) of specialised knowledge and specialised gazes on the world. A second assumption is that teacher education should prepare teachers to teach in schools, in terms of furthering their (1) professional knowledge, (2) professional practice and (3) professional engagement (Ingvarson, Beavis, and Kleinhenz, 2007). This implies (1i) knowledge about content and how to teach it, (1ii) knowledge about students and how they learn, (2i) curriculum, (2ii), classroom management, (2iii) assessment, (3i) reflecting on teaching, and (3ii) work with parents and others (*ibid.*).

By interrogating which factors are most related to learner performance according to international studies (next section), and regional/South African studies (subsequent section), questions to be considered in our teacher education programmes are formulated. Yet before I go there, I want to engage some issues of the aforementioned assumption.

Of course, how the first assumption above is interpreted is a crucial and critical issue. If it is taken in its narrowest form, it could be assumed to mean focusing on providing teachers with the content knowledge they will teach, and with a set of routines for transmitting it. In its broadest form, it could be taken to mean that teachers should be prepared to unpack the educational system and its positioning strategies, the psychology of the classroom, the recontextualisation of knowledge, and so forth, and be able to constantly reflect on and adjust their teaching accordingly – a critical stance perspective (Ainley and Luntley, 2007; Cochran-Smith and Lytle, 1999; Hiebert, Morris, Berk, and Jansen, 2007). In my claim that teaching is facilitation and organisation of systematic learning, I am driven to foreground providing learners with access to specialised knowledge above the critical stance. This does not reflect my own position as much as recognition of the need to start

somewhere, also considering that learning to teach from a critical stance may be too great a leap from our current situation (cf. Beeby, 1966). Yet it is a dimension which should be considered, and I will do so elsewhere.

There are obvious methodological as well as educational problems with attempting to assess learning by measuring performance through large scale tests. It is, however, the only measure we have which allows statistical comparisons and correlations with background factors. Smaller scale studies are obviously useful in exploring the causal relationships reflected in correlations and in considering competence rather than performance. In this paper, I have consciously and pragmatically taken the simplistic view and discuss effectiveness of teaching in terms of impact on learner performance, despite being aware of the shortcomings.<sup>1</sup>

Another issue is the extent to which teachers should be able to critically engage the selection of content to teach. As sociologists of education have emphasised repeatedly, it is not arbitrary what counts as knowledge (or art), and the selection of content to teach is itself a highly contested area, as is the extent to which the teacher should be a change agent (Liston and Zeichner, 1987). Again, it would be too far reaching to engage that in what remains a first investigation of a programme.

The exploration, and with it this paper, draws on a series of analytical tools. Some of these are drawn from work which assumes a sociological sensitivity to educational issues, and addresses positioning in relation to different types of knowledge. Others are based on existing and widely accepted (though also widely contested) categorisations of teacher knowledge linked to statistical analysis of which teacher actions impact on learner performance. In one respect, they are brought together in a way which can only be defended from a perspective of perspective pluralism (Skovsmose, 1990) or a claim to complementarity, in the same sense that descriptions of an electron as a particle or a wave are complementary. In another respect, what unifies them in this study is the extent to which they select, reproduce and position student teachers to do the same to their learners.

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<sup>1</sup> Such as ignoring more formative elements of teaching, or reducing the effect of caring relationships and the cumulative effect of learning which tends to disguise the work of previous teachers – a factor not often considered in impact studies. See (Lingard, Hayes, and Mills, 2003) for a discussion of related points.

## Factors related to learner performance according to international studies

International impact studies, most of which have been conducted in more developed contexts, indicate which factors explain differences in learner performance. Hattie and colleagues' summary of such studies suggest that learners' aptitude/ability account for half of the difference in performance; school, home, peers and the principal for about 20%; and the teacher for 30% (Hattie, 2003). The obvious problem with this list is not separating out socio-economic background. "The school is in fact the institution which, by its positively irreproachable verdicts, transforms socially conditioned inequalities in matters of culture into inequalities of success, interpreted as inequalities of talent, which are also inequalities of merit." (Bourdieu and Darbel, 1969/1991, p.111). Hattie only addresses this by saying that "the major effects of the home are already accounted for by the attributes of the student" (Hattie, 2003, p.2).

Hattie goes on to identify 16 characteristics of 'excellent' teachers, whose students perform better: they provide deep representations/content connections; a problem-solving stance to their work; they can anticipate, plan and improvise as required by the situation; are better at identifying important decisions; they are proficient at creating a learning classroom climate; have multidimensional perceptions of classroom situations; are more context-dependent; are more adept at monitoring and assessing students and provide more relevant feedback; are more adept at developing and testing hypotheses about learning and teaching; are more automatic; have high respect for students; are passionate; engage in enhancing learners' self-esteem and self-regulation; provide challenging tasks and goals for students; have positive influences on students' achievements; and enhance surface and deep learning.

Testing these in the USA, his paper claims that the three most important characteristics of teachers whose learners perform significantly better are: paying attention to deep representations, providing appropriate challenges to learners, and giving feedback which facilitates further learning. If teacher education is to take these findings seriously, it seems reasonable that these are the elements of teaching for which we should prepare our students. However, how does this relate to the other 70% of what impacts on learner performance, and how may the weighting of the factors be different in a developing context?

## What furthers learning most in South Africa?

There have been numerous studies engaging which factors are most strongly correlated with learner performance. Most recently, I have been involved in a medium scale study of grade 6 teaching and learning in one South African province.<sup>2</sup> Such studies must all be seen in the context of the very low achievement of the majority of South African learners as indicated by the TIMSS and other studies. In the SACMEQ<sup>3</sup> III study (Hungu, Makuwa, Ross, Saito, Dolata and Van Cappelle *et al.*, 2010), 44% of the learners in the province were found to operate on pre-numeracy or emergent numeracy levels, and this applied to even more of the learners in our study. The average score on the learner test in our study, which mostly consisted of grade 5 content, was just over 25% correct. All of this clearly indicates that the learners are operating far below grade level. Misconceptions are common (Christiansen and Aungamuthu, 2012).

The SACMEQ II study found substantial variations amongst schools in South Africa (SACMEQ II, 2010). This is in agreement with the findings from our own study which indicates that approximately 44% of the explainable variation in learner test performance can be attributed to differences between the classes, and since in most cases only one class from each school was included in the dataset, to differences between the schools. It appears to be a stronger factor than individual socio-economic status (Spaull, 2011; Van der Berg, Burger, Burger, De Vos, Du Rand and Gustafsson *et al.*, 2011), and our data also support this. This is clearly a consequence of the Apartheid system with its segregated population groups, where less educated teachers often teach in poorly equipped schools in poor communities.

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<sup>2</sup> While this was using the same instruments as the greater study headed by Carnoy and Chisholm (Carnoy, Chisholm, and *et al.*, 2008), the study was undertaken independently, as part of a project under the KwaZulu-Natal treasury headed by Wayne Hugo and Volker Wedekind.

<sup>3</sup> SACMEQ is the *Southern and Eastern Africa Consortium for Monitoring Educational Quality*, and it is a network of Ministries of Education. SACMEQ undertakes research to monitor, evaluate and inform improvements in education in the region. In 2007, data were collected in grade 6 classes to determine literacy and numeracy levels. Learners were selected in two tiered random process, first choosing schools and then learners, resulting in a sample of 9071 learners from South Africa. Some findings in this paper is from the previous SACMEQ study, three years earlier.

So while Hattie's (2003) comprehensive summary of the international literature indicates that the home factor is more important than the school location, the situation in South Africa may be opposite. While this could simply be because socio-economic status of the home is strongly linked to location, changing school appears to impact strongly on achievement (Van der Berg, *et al.*, 2011).

There are other important factors. One of these is language. This has been discussed extensively elsewhere (Setati, Chitera, and Essien, 2009), so for here it suffices to say that our research in the province indicates that learner misconceptions are more prevalent in African home language learners (Christiansen and Aungamuthi, 2012); and indeed it makes sense that basic interpersonal communication skills (BICS) in a second or third language does not imply the cognitive academic language proficiency (CALP) required to learn equally well from instruction in this language (Essien and Setati, 2007; Gerber, Engelbrecht, Harding, and Rogan, 2005; Setati, *et al.*, 2009).

That the school factor is very influential does not mean that the home situation does not matter. In our study, we also found that the amount of reading material the learners reported having in the home, having been read to as a child, reading at home, as well as who was their primary caretaker<sup>4</sup> had significant effect on test scores. The latter was also significantly related to the educational level of the caretaker. Preschool education is another factor which correlates with learner performance (Spaull, 2011).

## Aspects of teaching linked to learner performance

Within the school, more related to the teaching, important achievement factors appear to be: discipline/classroom management, feedback, frequency of

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<sup>4</sup> With fellow siblings and grandmothers as caretakers being associated with lower performance.

homework, feeling secure/safe in and around the school,<sup>5</sup> curriculum coverage, and presence of questions of high cognitive demand, while choice teaching methods such as direct instruction versus guided learning are less important— all according to our own study and others (Reeves, 2005; Spaul, 2011; Van der Berg, *et al.*, 2011). For reading, the availability of textbooks also makes a difference, but this does not apply to mathematics textbooks (Spaul, 2011).

So while a recent report states that “poor performance of teachers is a major reason for the poor performance of the South African schooling system” (Centre for Development and Enterprise, 2011, p.27), not only is this putting too much of the responsibility on teachers given the impact of previously mentioned factors, it must also be understood which performance aspects of teaching are most important. There are also indications that interventions in poorer schools do not have much, if any, impact – some schools do not manage to turn resources into an educational advantage, and the teachers’ content and pedagogical content knowledge has less impact the more disadvantaged the school is (personal communication regarding a review of school interventions, Paul Hobden). Along the same lines, strong Kenyan learners improved their performance substantially when they were given textbooks, whereas the same did not apply to weak or even average learners (Glewwe, Kremer, and Moulin, 1998).

To deepen our understanding of this, we engaged detailed analysis of the video recordings of teaching in our study. The study focused on grade 6 teaching and learning, at 39 schools chosen through stratified random sampling within one district in KwaZulu-Natal. A full set of data from a school consisted in a teacher questionnaire and a teacher test on content knowledge and some pedagogical content knowledge (PCK), a learner questionnaire, a learner test from the start of grade 6, a learner test from the

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<sup>5</sup> In our study, learners not feeling safe in school was statistically significant in relation to their test scores. 39% of the learners in the final sample, ie learners who sat for both tests, were sometimes scared of being hurt by other learners, and 4% often; 36% were sometimes scared of being hurt by a teacher in the school, and 4% often. An alarming number of learners do not feel safe in the school environment. Violence amongst learners and by teachers may not be uncommon, but learners also fear violence from the surrounding community, such as rape or abduction on the road to and from school; in our study, 36% sometimes felt scared of being hurt by someone from outside the school, 7% often. This is likely why a solid demarcation of the school area is one of the characteristics of resilient schools (Christie, 2001).

end of grade 6, a principal questionnaire, a video recording of a lesson in each grade 6 class, and a sample of learner workbooks to give some indication of curriculum coverage. As the data collection was difficult, we did not always get a full set of data from all schools, which limits the validity of our study. Yet the preliminary analysis indicates some issues for concern. And these made us ask how great a factor the teacher is in South African learner achievement, and what aspects of teaching and teacher knowledge seem to matter the most. All of this would inform what we foreground in our teacher education programmes.

The video recordings were analysed for the practical PCK displayed, in particular the extent to which the teacher linked to prior knowledge, encouraged longitudinal coherence, showed more than one approach to a problem type, and identified and addressed errors or misconceptions (Ramdhany, 2010). They were also analysed for the extent to which teachers provided opportunities to develop mathematical proficiency<sup>6</sup> (Ally, 2012). All of these factors were then related to the teachers' test scores and the learners' achievement. I will briefly discuss the findings, as they informed the formulation of questions for the interrogation which informed this paper.

The teachers generally performed poorly on the test they were given. Though we only had both questionnaires and tests from 34 teachers, we tried to see if there was any significant difference in performance on the test depending on educational levels: the ANOVA showed no significant difference between teachers with a formal and no formal teacher qualification, respectively. This agrees with the finding from the SACMEQ study which showed that only about 6% of the variation in teachers' scores on a learner test was related to qualifications.

Teacher content knowledge is often considered crucial, but the studies indicate that it accounts for little of the variance in learner performance. For instance, an analysis of the SACMEQ III data indicates that a 100 point – or roughly one standard deviation – improvement in teacher score only raises learner

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The notion of mathematical proficiency contains five dimensions: conceptual understanding, procedural fluency, adaptive reasoning, productive disposition and strategic competence (Kilpatrick, Swafford, and Findell, 2001). Ally distinguishes between mathematical proficiency, which focuses on the learning, and opportunities to develop mathematical proficiency (OTDMP), which focuses on teaching. Further development of indicators of levels of OTDMP is needed, but the approach looks promising.

scores by 4.8 points; and this impact is much smaller in the poorer quintiles of schools (Spaull, 2011).

There were differences in the extent to which the teaching displayed practical PCK, and the teachers who displayed more PCK were more likely to have a professional teaching qualification or an academic qualification, but the results were not statistically significant (Ramdhany, 2010, p.65). More depressing, there was no significant difference in learner achievement gains from test 1 to test 2 between the teachers who displayed more PCK and those who displayed less (Ramdhany, 2010, p. 65). This is not necessarily the case in other contexts. Thus, a German study found that “when their mathematics achievement in grade 9 was kept constant, students taught by teachers with higher PCK scores performed significantly better in mathematics in grade 10” (Krauss and Blum, forthcoming). Thus it remains to be seen if pedagogical content knowledge matters more in high performing South African schools, where learners have attained a basic level of comprehension and competency – and are not hungry or in other ways struggling to have their basic needs met.

All in all, the opportunities to develop mathematical proficiency in the classes in our study were limited: only in half of the 30 classrooms from which videos were coded for this aspect, was conceptual understanding facilitated, and in 76% of these cases, a concept was simply stated or learners’ attention directed to it. In the remaining 24% of the cases, a concept was formulated through discussion or demonstration and mathematically supported by teachers or learners (Ally, 2012). Only in about a third of the cases were opportunities provided which clearly clarified the concept with explicit links made to other concepts (Ally, 2012). This is not the best quality of teaching, if I go by the claim by Hattie that ‘deep representations’ is central to good teaching, as well as the stressing of conceptual understanding in much mathematics education literature from the past decades.<sup>7</sup> We did find a significant correlation (at the 5% level) between the highest secondary school qualification obtained by the teachers, and the extent to which they provided their learners with the opportunity to develop mathematical proficiency, in particular conceptual understanding. There was some indication that there was a correlation between learner achievement gains and the opportunity to develop adaptive reasoning yet this is not a reliable correlation since this strand was only evident in 9

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<sup>7</sup> This of course should not be taken to mean that conceptual understanding should be pursued without procedural fluency etc. though this perception may be prevalent (Bossé and Bahr, 2008).

lessons in the study (Ally, 2012). Yet, this result is in agreement with the claim by Kilpatrick et al, that adaptive reasoning is “the glue that holds everything together” (Kilpatrick, *et al.*, 2001, p. 129).

While this seems to indicate that teacher education does not make much difference in South Africa, we must remember – besides the validity issue from the small number of teachers who actually provided us with all the relevant data – the role of the contextual factors mentioned above. Thus, it appears that school location and home factors matter much more in South Africa than in the studies to which Hattie refers, while teaching quality is generally questionable and therefore the impact of this factor is dwarfed by the general conditions of schooling and living in most of our communities.

What does this mean to mathematics teacher education? I engaged our local programme as a case.

## Questions for the local teacher education programme

The Post Graduate Certificate in Education (PGCE) is a one-year post-graduate teaching qualification. Students with a bachelor’s degree are accepted into the programme if they meet the entry requirements for their desired disciplinary specialisation. For mathematics teachers up to grade 9, this means having completed more than one course of first year university mathematics, and for teachers for grade 10–12, some second year university mathematics is required. The entry requirements are lower than in many programmes internationally, and this must be seen in the light of the severe national shortage of teachers of mathematics.

The programme comprises eight courses: 3 general education courses, 2 practical courses spent in schools (‘teaching practice’), and 3 courses specific to their choice of discipline (teaching specialisations). There are two mathematics education courses available, one for grade 7–9, and one for grade 10–12. Both have as their stated purpose to equip the students with PCK for teaching the content specified by the national curriculum.

The discussion of what South African teachers need seems to indicate that they must be prepared to deal with the broader context to further the resilience of the school (Christie, 2001), which may imply taking some form of teacher

leadership role and extending the teaching to include a stronger ethics of care (Grant, Jasson, and Lawrence, 2010). In the context of literacy in pre-school, Prinsloo and Stein go as far as to suggest that growing a vegetable garden is part of literacy education (Prinsloo and Stein, 2004). In any case, it is important that new teachers understand these issues and their impact on education. Thus, I was interested in seeing to what extent contextual issues were engaged in our PGCE programme.

For teacher education to counter the low educational achievement of our learners, it is important that its recognition and realisation rules are within specialised knowledge, not allowing students to rely first and foremost on their beliefs, past experiences, etc. For this purpose, I engaged the extent to which our teacher education programmes reflect a knowledge legitimation code (Maton, 2006), where claims to legitimacy are situated within the epistemic rather than the social relation – discussed further in the section on analytical tools. Yet students must be provided access into specialised knowledge, and thus it was important to consider how theory and practice were related in the courses. I did so by considering the semantic gravity of the course (Maton, 2009).

Next, I interrogated the extent to which the content of the courses reflected the indicators of good teaching outlined by Hattie (Hattie, 2003), and how this related to the context just outlined. Seeing that opportunity to develop adaptive reasoning may be linked to learner achievement gains, I also looked for the extent to which preparation for this was foregrounded in our programme.

The issue of *how* to convey these issues to the students were not considered presently.<sup>8</sup>

## Analytical tools

In the study, I operationalised a number of coding systems. First, I considered

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<sup>8</sup> There is much input on this, see for instance (Adler and Davis, 2006; Artzt, 1999; Jaworski and Wood, 1999; Langford and Huntley, 1999; Liston and Zeichner, 1987; Lloyd, 1999; Nakanishi, undated; Ohtani, 2003; Parker and Adler, 2012; Rayner, Pitsolantis, and Osana, 2009; Schifter, 1998; Yang and Leung, 2011). In the South African context, a refreshing perspective is still delivered by (Breen, 1991/2).

five knowledge domains: Contextual knowledge including knowledge of learners' background; curriculum issues, including meta-questions regarding the nature of teacher knowledge and questions regarding the overall purposes and goals of education; content knowledge; PCK; and general pedagogical knowledge. Other contributions to this issue have discussed the problems of identifying the knowledge domain of a task or question, in particular in relation to PCK (Adler and Patahuddin, forthcoming; Krauss and Blum, forthcoming). In order to work around this problem, in a pragmatic way given the preliminary nature of this study, I did this by working with some more clearly demarcated sub-categories. Another consideration was that, unlike the questions discussed by (Adler and Patahuddin, forthcoming), the tasks comprising the data in this study did not have to address one knowledge domain only, so it was possible for a question to fall within more than one category or sub-category.

Content knowledge was separated into three categories, namely declarative (conceptual), procedural, and 'doing mathematics' (Stein, Smith, Henningsen, and Silver, 2000). Some of Hattie's (2003) 16 characteristics of excellent teachers would be hard to assess outside of the practice of the classroom, but I felt that nine of them could, and these were then grouped either as PCK or as pedagogical knowledge. Thus, PCK was given five sub-categories:<sup>9</sup> assessing and giving feedback; deep representations including linking to other content; focusing on key aspects of teaching; challenging the learners; and focusing on deep rather than surface learning. Likewise, pedagogic knowledge was distinguished as: facilitating a classroom culture conducive to learning; furthering a multi-dimensional understanding of the classroom; having a pedagogical repertoire with more than one informed approach; and adapting a problematising strand where more information is sought before making a teaching decision. These sub-categories were also not mutually exclusive.

Second, I focused on legitimisation of knowledge. As has been pointed out by Adler and colleagues, the way knowledge is legitimised in education can be revealing. For instance, (Parker and Adler, 2012) found that a mathematics lecturer legitimised content knowledge by referring to mathematics, but knowledge about teaching was legitimised with reference to mathematics, the

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<sup>9</sup> This does not imply that these are necessarily the best descriptors or indicators of PCK, nor that they are exhaustive. I used them because they are important factors determined in Hattie's work, and because they relate well to previous work on the nature of PCK, which I will not go into here.

lecturer's personal experiences, etc. Working with examination papers does not allow me to follow Parker and Adler's method. I turn instead to Maton, who is interested in the legitimisations of knowledge. He postulates that "knowledge comprises both sociological and epistemological forms of power" (Maton, 2000, p.149), and "knowledge claims are simultaneously claims to knowledge *of the world* and *by authors*" (ibidem, p. 154, italics in original). Two modes of legitimation are worked with, a 'knowledge mode,' and a 'knower mode'. In the knowledge mode, the epistemic relation, the knowledge, is foregrounded. In the knower mode, knowledge is more specialised by its social relations, whether it is who you are or your particular cultivated gaze on the world that counts. To distinguish the legitimation code, I drew on the analytical categories presented by Chen, Maton and Bennett (2011). They distinguish between four:

- knowledge code (ER+, SR–), where possession of specialised knowledge, procedures or skills is emphasised as the basis of achievement and the dispositions of actors are considered less significant;
- knower code (ER–, SR+), where the dispositions of actors are emphasised and specialist knowledge or skills are downplayed. These dispositions may be considered innate or natural (e.g. notions of 'genius'), cultivated (e.g. an artistic sensibility developed through immersion in great works), or socially based (e.g. a specific gender);
- elite code (ER+, SR+), where achievement is based on having both specialist knowledge and being the right kind of knower; and,
- relativist code (ER–, SR–), where neither specialist knowledge nor particular dispositions are important (Chen, *et al.*, 2011, pp.131–132).

The third tool for analysis engaged the extent to which knowledge in our programme was context dependent or abstract. Whereas Bernstein made a distinction between horizontal and vertical discourses in the knowledge production sphere (Bernstein, 1999), Maton is also interested in the reproduction of knowledge and employs a more graduated measure which will allow him to see if a curriculum favours segmented or cumulative learning (Maton, 2009). One measure which assists in this is semantic density, which indicates the extent to which meaning depends on context – weaker semantic gravity is associated with vertical discourse and more context independence,

stronger semantic gravity with horizontal discourse and more context dependence (*ibid.*). As Dowling discusses, it is necessary to start from less specialised knowledge to gain access to the specialised, and the specialised can then be applied (Dowling, 1998). In other words, there will be a gradual decrease of context dependence as learners move from the everyday into mathematics, and then an increase in contextualisation as mathematics is applied. Maton has picked up on the move towards more abstraction in his 2009 paper, which develops categories of analysis for abstraction. He does not have a similar set of categories for when students are asked to apply more decontextualised knowledge. I therefore developed 6 categories for abstracting and 6 for applying, paired according to increasing degree of decontextualisation. I also added a category for entirely decontextualised, that is, theoretical, knowledge. The coding categories are shown in the appendix, which also compares them to Maton's categories, the extended Bloom's taxonomy (Anderson, 2005; Anderson and Krathwohl, 2001), and (Hatton and Smith, 1995)'s types of reflection in teacher education.

## Method

The current results are from the first, exploratory phase of the study only. For the purpose of interrogating the extent to which we prepare teachers appropriately for teaching mathematics in the South African context, I decided to start by interrogating the evaluative rules of the PGCE maths programmes, starting with what the exam papers imply are core knowledge areas and competencies. All papers from the three general education courses (610, 620 and 630) and from the two mathematics education courses (GET for up to grade 9 and FET for grade 10–12) were coded in NVivo 9. When comparing the exam papers, the mark allocation was taken into account. This was challenged by the fact that all but one paper allowed students to choose between questions, however I felt that the entire paper had to be considered in determining the knowledge legitimised in the programme.

Firstly, all questions were coded for their knowledge domain and sub-categories, as previously listed. These codes were not exclusive, as one exam question could relate to several categories.

To determine the legitimisation code of a question, an exam question was coded as dominated by a knowledge code when specialised knowledge was clearly

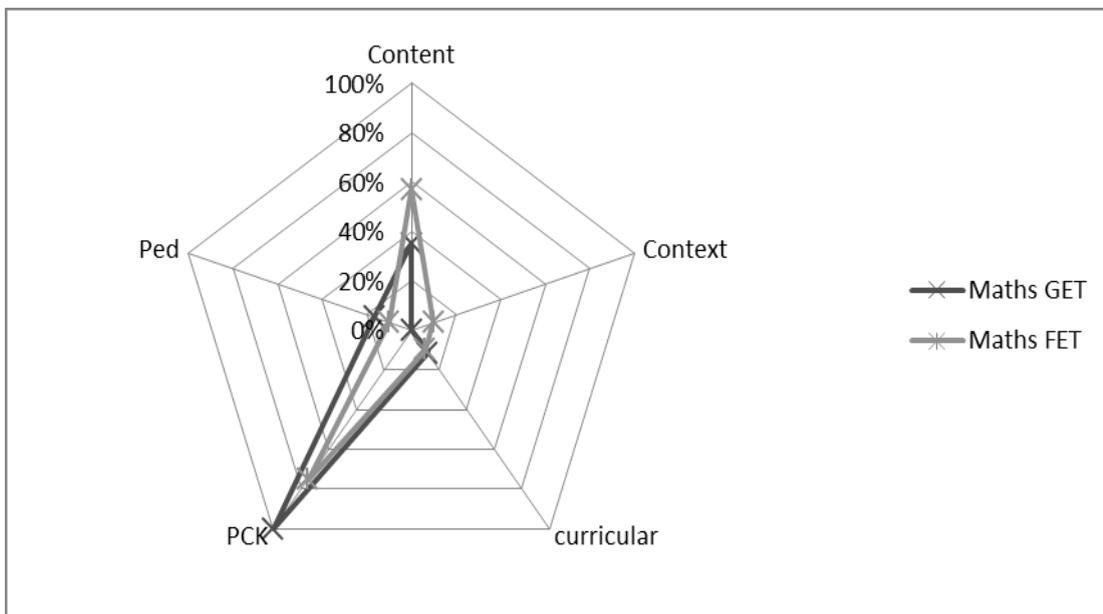
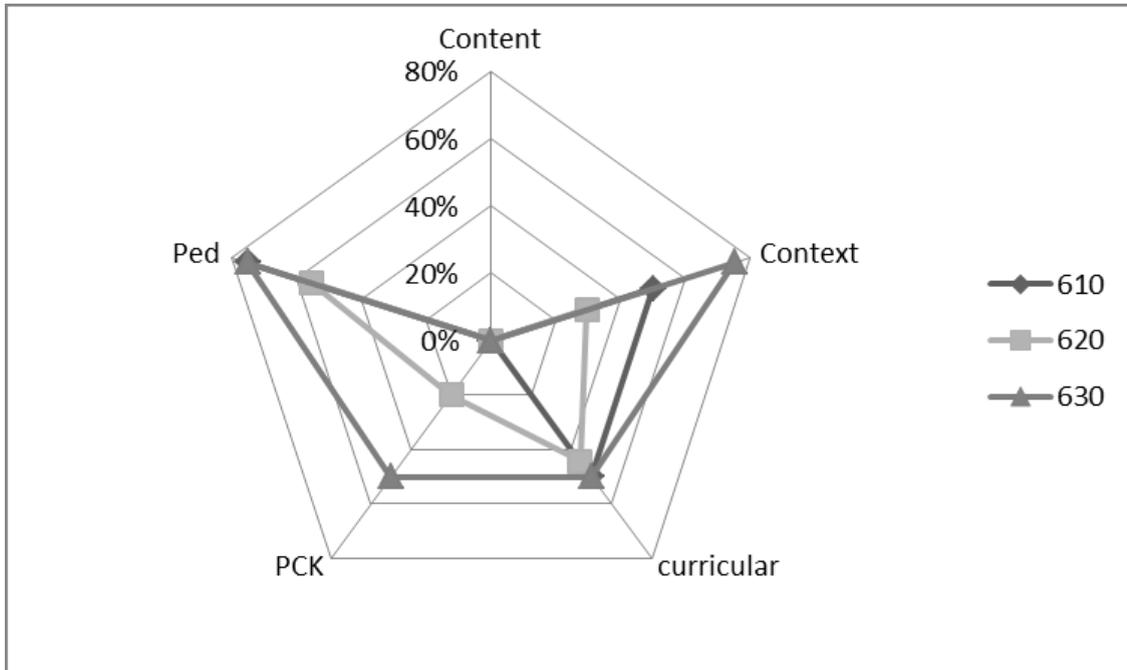
required to answer the question, even if there was some allowance for a social gaze with reference to students' experiences, beliefs and attitudes not informed by specialised knowledge. An exam question was coded as being within an elite code when students were asked to engage their beliefs, experiences etc. in the light of specialised knowledge or vice versa. Finally, the knower code was used when students were asked for their individual beliefs and opinions with little or no indication that specialised knowledge must be engaged. These codes were mutually exclusive.

Finally, each question was coded for its degree of semantic gravity, and next for whether it encouraged students to abstract (weaken semantic gravity) or apply (strengthen semantic gravity).

## Findings

It was not unexpected that the knowledge categories assessed strongly depend on whether the course is a general education course or mathematics education course, with the latter foregrounding PCK and mathematical knowledge for teaching:

**Figure 1: The five knowledge dimensions in the courses in the programme. Figure 1a shows the knowledge dimensions in the general education courses; Figure 1b the knowledge dimensions in the mathematics education courses.**

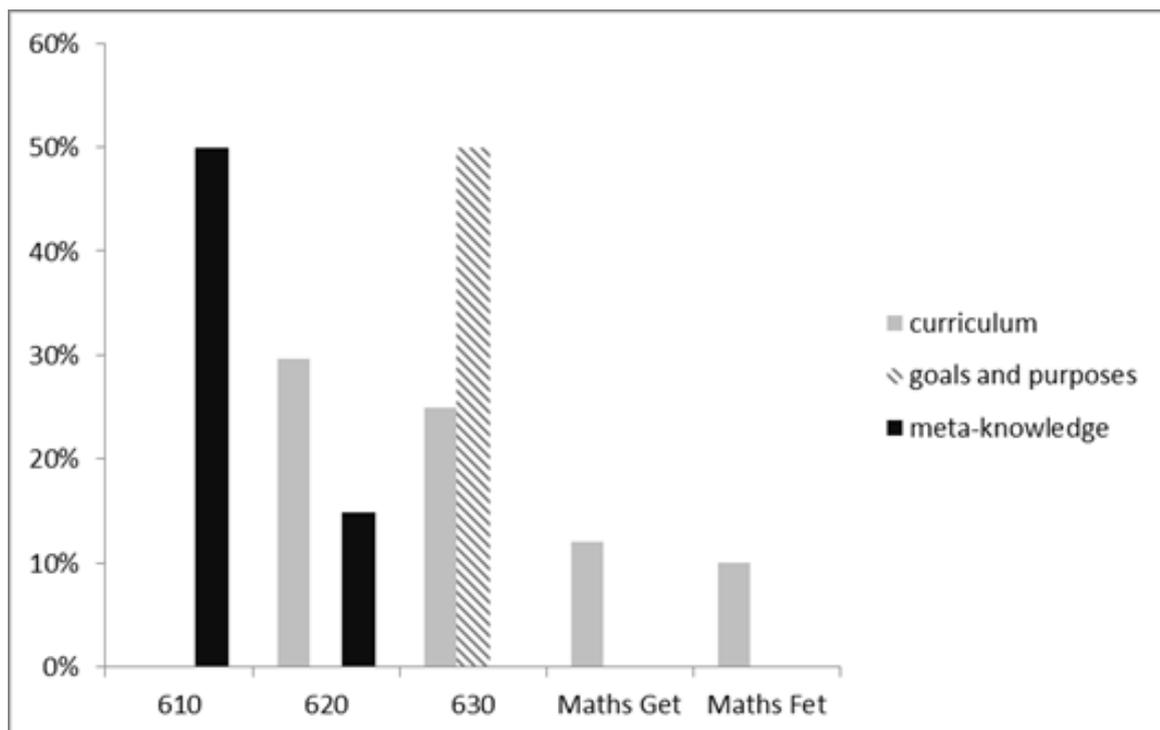


It is unclear how integrated the knowledge dimensions should ideally be, but in my view students would benefit from leaving their teaching education with a coherent message about what is important in teaching. It warrants further

interrogation about the extent to which the programme as a whole achieves this, across the knowledge domains. As discussed earlier, the broader context of education in South Africa has an immense impact on the teaching-learning situation, and appears to be well addressed in the general education courses, but only figures in passing in one of the two mathematics education examination papers. I need to interrogate further the extent to which the issues showing up in our grade 6 study correspond to what is engaged in the courses, for instance if engaging issues of leadership, language, relating to the home, etc. are addressed. If contextual issues are engaged in too decontextualised or general ways, students may not develop practical skills in handling the situation in struggling schools. Indeed, one student said that the university trains student teachers for ‘A schools’, but force them to also teach in ‘B schools’ (Mari van Wyk, presentation to faculty, November 2011).

The curriculum also appears more taken for granted in the mathematics education examinations, which do not engage the goals of education at all:

**Figure 2: The percentage of questions in the five exam papers which engaged with three sub-categories of curricular issues in the five courses in the programme: unpacking the curriculum, engaging goals and purposes for mathematics education, and meta-knowledge about what teachers should know and be able to do.**



Thus, the mathematics courses appear detached from the broader context. In itself, this is not a problem – clearly, it facilitates specialised knowledge in mathematics and the PCK of mathematics – but the programme as a whole needs to be clearer on whether there are benefits to making links between knowledge domains and if so how this is best accomplished.

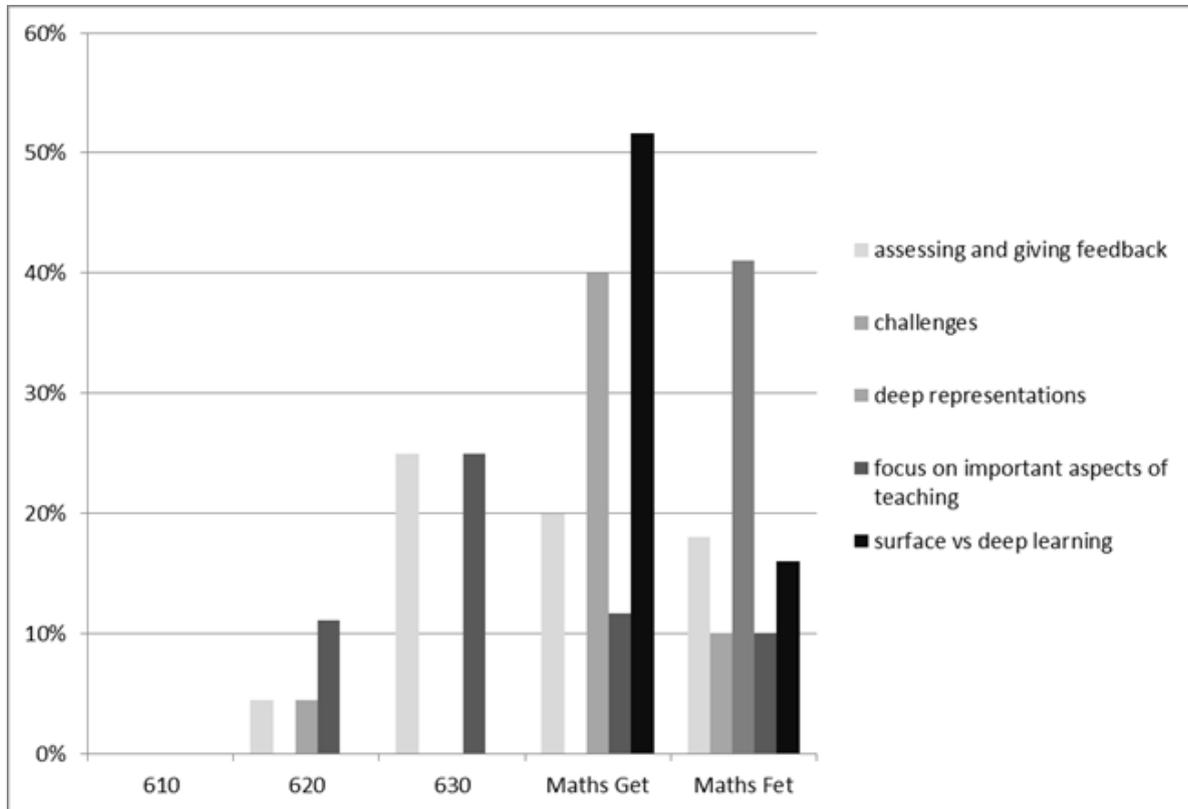
Looking at questions across the papers which had been coded within more than one knowledge domain, I looked at the frequency of overlapping codes (see Table 1). It was clear that content knowledge was most likely to occur in the same question as PCK, that contextual knowledge was overlapping most with curriculum issues and general pedagogy, and that pedagogical content knowledge occurred more often together with PCK. This indicates a decent degree of connections, but the nature of these must be further investigated.

**Table 1: Frequency of overlaps of knowledge categories in the five courses. PCK and content knowledge were both assessed in 10 questions.**

|                      | Content knowledge | Contextual knowledge | Curriculum issues | PCK | Pedagogic knowledge |
|----------------------|-------------------|----------------------|-------------------|-----|---------------------|
| Content knowledge    | –                 | 0                    | 1                 | 10  | 0                   |
| Contextual knowledge | 0                 | –                    | 7                 | 1   | 3                   |
| Curriculum issues    | 1                 | 7                    | –                 | 3   | 2                   |
| PCK                  | 10                | 1                    | 3                 | –   | 9                   |
| Pedagogic knowledge  | 0                 | 3                    | 2                 | 9   | –                   |

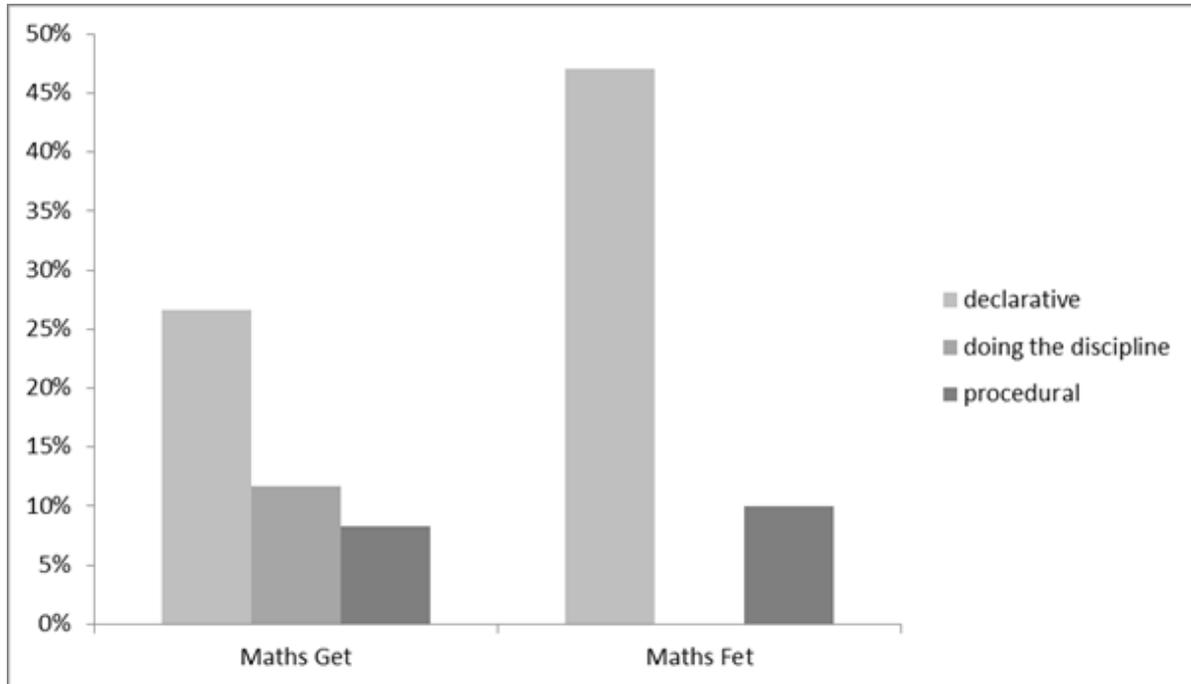
When it came to PCK, the mathematics education courses covered a broader range of aspects, though not with equal emphasis:

**Figure 3: Frequency of the sub-categories of PCK in the five courses**



From the perspective of Hattie's findings (2003) and in the light of the limited extent to which opportunities to develop conceptual understanding were found in our grade 6 classrooms, it is comforting to me to see the strong emphasis on deep representation in the two content courses. Yet, the absence of this dimension in the general education courses is ground for concern – where it could have been engaged in a more decontextualised way as a priority. The focus on the conceptual dimension was also clear in the mathematics for teaching courses, whereas it appears that the engagement with the one opportunity to engage in deriving a formula was limited to considering how to show this to learners, and thus did not appear to stress that teaching mathematics can include providing opportunities to develop adaptive reasoning:

**Figure 4: The types of content knowledge assessed for in the two mathematics education modules in the programme**



Returning to PCK, the second of the three most important types of teacher action, according to Hattie, is assessing and giving feedback, and this aspect is addressed in the general education courses, increasing with each course, and also assessed in the mathematics education courses. However, a more detailed reading of the exam questions indicates that questions in this category focus on identifying learners' level of understanding and not on how to give appropriate feedback. Only one question in 630 asks "Bearing the theories in mind, describe how teachers in South Africa can help to facilitate . . ." and this does not assess the complex process of giving feedback in any substantial way (cf. Hattie and Timperley, 2007). Thus it is clear that in this crucial respect, we are failing in the preparation of our mathematics teachers. This is more reason for concern when I note that feedback in the observed grade 6 classrooms mostly is what has been called task feedback (Hattie and Timperley, 2007) – simply indicating if an answer is correct or incorrect – and often is tacit.

The third of Hattie's most important categories is challenging learners. This is also notably absent, with only the one mathematics education course assessing this aspect, and only in a very indirect way. This question reads:

You want the learners to draw a histogram based on the data. You are thinking about whether you should give them the relevant intervals or ask them to group the data using their own choice of intervals. There are both several weak and several strong learners in the class. Discuss some advantages and disadvantages of each of the options.

The students are being asked to consider two options in relation to the mixed ability class, but there is certainly no explicit requirement that they consider how the task could be made to be challenging for the learners (albeit this consideration was discussed in the marking memorandum). Thus, there is no indication that the courses have engaged in preparing the students to teach large classes with learners of mixed ability, so that learning can happen for more learners irrespective of their current level. One intervention which took a strict mastery approach to mathematics learning in primary school (Schollar, 2008) indicates that such careful challenging of the learners on their current level can be highly beneficial. It is also a pressing issue in the light of the span of abilities in many South African classrooms, in particular in rural contexts. Thus, our teacher education programme seems to fall short in this respect as well.

With respect to pedagogical knowledge, I expected this to be more prevalent in the general education courses. It was noteworthy that the dimension of creating a classroom conducive to learning was not assessed in the mathematics education courses, which seems concerning to me as it detaches considerations of content learning from overall classroom environment. In this category, I included allowing learners to make mistakes, and with the vast body of research on learner misconceptions in mathematics education (Ben-Zvi and Garfield, 2004; Cuffel, 2009; Molina, Castro, and Castro, 2010; Walcott, Mohr, and Kastberg, 2009) as well as socio-mathematical norms (Cobb, 1991; Cobb and Bauersfeld, 1995), I find it highly surprising that this was not foregrounded in the assessment in the courses. Interestingly, general pedagogical skills which some of our students feel they have not adequately developed, are how to use textbooks and the chalkboard well (Mari van Wyk, presentation to faculty, November 2011). Yet textbooks are only effective learning resources if they, besides the content being appropriate and well organised, are used well. None of the questions in the exam papers engaged how to judge the quality of a textbook or even a task, though previous papers had asked students to determine the van Hiele geometrical level of tasks, for instance. One question asked students to identify to what extent a textbook

task fell within the mathematics literacy<sup>10</sup> curriculum, but other than that, no questions examined students' ability to engage with teaching resources in any respect. It seems likely, then, that this is assumed to be a skill which students possess or develop, without explicit teaching, or that it is something they will 'pick up' while on teaching practice in schools.

With respect to the legitimation code, all courses were strongly dominated by either a knowledge or an elite code, with only the 610 course having a small occurrence of the knower code. Separating the knowledge code into a strong and a less strong knowledge code ('knowledge' for short in the table below), it appeared that the legitimation code varied somewhat with knowledge type:

**Table 2: Number of exam questions coded as exhibiting knowledge or knower codes in the five knowledge categories across the courses in the programme**

|                  | Content knowledge | Contextual knowledge | Curriculum issues | PCK | Pedagogic knowledge |
|------------------|-------------------|----------------------|-------------------|-----|---------------------|
| Strong knowledge | 8                 | 0                    | 1                 | 10  | 1                   |
| Knowledge        | 2                 | 1                    | 2                 | 7   | 5                   |
| Elite codes      | 2                 | 7                    | 6                 | 10  | 13                  |
| Knower           | 0                 | 0                    | 0                 | 0   | 1                   |

Thus, the content knowledge is dominated by a very strong knowledge legitimation code, which is perhaps expected given the nature of mathematics as a discipline. Pedagogical content knowledge is more strongly dominated by a knowledge code than pedagogical, context and curriculum knowledge. With the current crisis in education, this is in need of further exploration, because it would be problematic to assume that students develop a trained gaze as a teacher after just four years of teacher education, in particular if the gaze encouraged or assumed at university is at odds with the views of teaching students bring from their own experiences.

The semantic gravity of the courses varied somewhat, with the general education courses being more decontextualised from direct experience and the context of application of the knowledge. The relatively context dependent

<sup>10</sup>

Mathematics Literacy is a school subject in South Africa, for grades 10–12.

activity of providing examples of categories within given frameworks such as the Van Hiele levels dominated the mathematics education courses, with relatively weak semantic gravity tasks such as discussing/critiquing and generalising/combining being more prevalent in the general education courses. The five courses were almost entirely dominated by applying decontextualised knowledge, in contrast to abstracting general principles from contexts. This could be explained with reference to the professional orientation of teacher education programmes, but it remains an open question to what extent this is the best approach to learning relevant knowledge and competencies, as well as the best approach to assessing them. One way to engage this further is to see to what extent the mathematics of the mathematics education courses is being 'unpacked' (Adler and Davis, 2006), but I believe we need a similar notion for other knowledge domains.

## Conclusion and discussion

In this explorative first phase of the study, I contrasted national/regional with international findings about factors most strongly correlated with learner performance. I used this to develop tentative indicators reflecting aims for our mathematics education programmes in preparing students to teach in South Africa. I also considered to what extent specialised knowledge was foregrounded in a PGCE programme, meaning that students do not rely on their own beliefs and experiences. In this phase, the evaluative rules of the UKZN PGCE programme were analysed in order to determine what knowledge is legitimised or valued in the programme.

It was found that the programme is strongly within a knowledge code, though with some space given for the students to relate their own experiences etc. even in the examination. The semantic gravity varies, but generally focuses on applying more decontextualised learning. It remains a question for further exploration if this would benefit from being supplemented by more work on abstracting and generalising.

The programme covers many relevant aspects, but it needs further exploration in terms of how well they are linked and to what extent they provide a connected and complete teacher education. In particular, it must be considered to what extent the contextual knowledge dimension is linked to the

disciplinary and pedagogic practice of teachers or operates on a more general level.

With respect to pedagogical content knowledge in particular, I found that the programme strongly legitimises teaching for deep representations, and also addressed the assessment of learners, but offers little direction on how to challenge learners on their level, and on how to provide substantial and constructive feedback to learners. In addition, no attention was given to the development of skills related to provide learners with the opportunity to develop adaptive reasoning.

In addition, it was found that some aspects which may not have been considered by Hattie, who compared experienced and expert teachers, nonetheless could be crucial to include in a pre-service programme. In particular the issue of preparing students to use textbooks and other materials appropriately needs to be considered.

The overall approach of comparing what we teach in the teacher education programmes to what we find works in our schools is crucial, and the sub-categories of pedagogical knowledge and pedagogical content knowledge derived from Hattie's work are deemed promising, though they may need adjusting to reflect some taken-for-granted aspects. Bringing in Maton's modes of legitimation (knowledge and knower codes) assisted me in assessing the extent to which the courses relied on an implicit gaze or foregrounded the epistemic, and I think this provides a useful insight into our teacher education programmes. So did using semantic gravity, but it was more the direction of the move between high and low semantic gravity that provided a useful insight in the relation between theory and practice in the course, than being able to determine the semantic gravity itself.

All in all, this can only be the beginning of the development of a more powerful and theoretically coherent language of description with which to assess what we do in teacher education.

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|   |        |   |  |  |   |
|---|--------|---|--|--|---|
| Semantic degree of independence from context of experience or use | Weaker | Allen's framework used by Maton, 2009 (about case studies)  | Hatton & Smith, 1995 on reflection in teacher ed   | Extended Bloom's taxonomy  | Criteria for evaluating assessment tasks: what does the task require? What are the implicit recognition and realization rules?  |
|   |        | <p><b>Abstraction:</b> Presents general principle or procedure to address wider or future practice</p> <p><b>Generalisation:</b> Present a general observation or draws generalize conclusion about issues and events <i>in</i> the case</p> <p><b>Judgement:</b> Goes beyond representing or interpreting to offer a value judgement or claim</p> <p><b>Interpretation:</b> Seeks to explain a statement by interpreting information from the case or adding new info. May use litt or personal experience</p> <p><b>Summarising description:</b> Descriptive that summarises or synthesizes information from the case, but with no new info</p> <p><b>Reproductive description:</b> Reproduces information directly</p> | <p>seeing as problematic, according to ethical criteria, the goals and practices of one's profession</p> <p><b>Dialogic:</b> weighing competing claims and viewpoints and exploring alternative solutions</p> <p><b>Descriptive:</b> seeking what is seen as 'best possible' practice</p> <p><b>Technical:</b> drawn from a given theory base, but always interpreted in light of personal worries and previous experience</p> | <p><b>Creating</b> it the process of putting together elements to form something else, sometimes a new creation but it could also be the recreation of something that has been produced before</p> <p><b>Evaluating</b> is the process of making judgements normally based on some kind of criteria or a set of standards</p> <p><b>Analyzing</b> is the ability to break down a problem into its parts and see how the parts interrelate and who you can use them together</p> <p><b>Applying</b> is carrying out a procedure in a given situation, which is different from the knowledge of procedures spoken about earlier although the two are often linked as you need the knowledge of a procedure to apply it correctly</p> <p><b>Understanding</b> is the process of constructing meaning from various sources of information</p> <p><b>Remembering</b> is the process of retrieving knowledge from memories</p> | <p>Applying ('down')</p> <p>Entirely abstract: Theoretical work only</p> <p><b>Abstract:</b> Asks student to relate a general principle from linking the issues of the question with information and in particular theory or theoretical concepts</p> <p><b>Generalize:</b> Asks the student to combine the given fact and information from other sources in presenting a more generalized conclusion</p> <p><b>Critique:</b> Asks the student to make claims about a stated general issues reflected in a situation to which the question refers; discuss a number of given factors</p> <p><b>Interpret:</b> Asks the student to explain the situation referred to in the question through reference to the literature</p> <p><b>Summarize:</b> Asks student to summarise key issues from her/his experience or from the specific situation referred to in the question</p> <p><b>Describe:</b> Asks student to engage direct experience or to respond to a specific situation without reference to broader context or principles;</p> |
| stronger  |        |   |  |  | <p>Abstracting ('Up')</p> <p><b>Expand:</b> asks student to relate theoretical concepts to each other in a principled way, for instance by drawing out a new aspect, but based on a specific context or class of contexts</p> <p><b>Combine:</b> asks student to link more than one theory to the situation of the question</p> <p><b>Discuss:</b> distinguish or compare concepts or theories or approaches, discuss conditions for something to be applied/implemented</p> <p><b>Characterise:</b> describe, classify or characterize something using an existing framework</p> <p><b>Exemplify:</b> give specific informed/reasoned example of something within a stated category; explain how would carry out a specific principle in practice</p> <p><b>Illustrate:</b> simply state an example of something</p>   |

Appendix

# Journal of Education

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The *Journal of Education* is an interdisciplinary publication of original research and writing on education. The Journal aims to provide a forum for the scholarly understanding of the field of education. A general focus of the journal is on curriculum. Curriculum is understood in a wide and interdisciplinary sense, encompassing curriculum theory, history, policy and development at all levels of the education system (e.g. schooling, adult education and training, higher education). Contributions that span the divide between theory and practice are particularly welcome. Although principally concerned with the social sciences, the journal encourages contributions from a wider field.

While it is intended that the journal will remain academic in nature, the readers are considered to be educational generalists and articles which are of interest to such readers will receive preference. Potential contributors are asked to ensure that submissions conform to the guidelines outlined at the back of the journal.

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Articles and review essays are reviewed by anonymous external referees. Appropriate papers will be refereed for significance and soundness. Papers are accepted on the understanding that they have not been published or accepted for publication elsewhere.

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*Journal of Education* style of referencing is a requirement. References in the text should appear as follows:

No country in the world can afford the schooling its people want (Reimer, 1971) and it has been argued that “of all ‘false utilities’, school is the most insidious” (Illich, 1971, p.60).

The references should be listed in full at the end of the paper in an acceptable standard format, preferably the following:

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Surname(s), Initial(s). Year of publication. Title of article. *Name of magazine or newspaper* day and month: inclusive (and additional) page numbers.

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Surname(s), Initial(s). Year. Title: additional title information. Description of occasion (including the nature and subject of the conference or meeting, name of the society or group, the place at which it was held and the date(s) on which it was held).

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*Most journals now have a per page fee which contributors are required to meet should their articles be accepted. Does the Journal of Education levy such charges?*

Yes. This step was necessary to cover the costs of the increased number of issues each year. A levy of R100 per page will be applied to successful articles submitted to our office. The central research offices in most institutions of higher education routinely arrange for such payments to be made. We encourage individual authors who do not have such cover to contact us.

*Are articles peer reviewed?*

Yes. Our goal is for articles to be refereed by three experts in the field.

*What is the waiting period after submission?*

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Yes. The electronic version of the article should be sent as an email attachment.

*To what extent should an article being submitted be presented in ‘the style’ of the journal?*

Citation and referencing should be in the style of the journal (see the previous section ‘Notes for Contributors’). Authors are not expected to reproduce the particular fonts and font sizes used in the journal, but the levels of headings and subheadings should be clear. With regard to the electronic version of the article, we prefer as little formatting as possible.

*Does the journal have a policy to encourage and support budding novice researchers?*

Unfortunately not – this is simply beyond our capacity. While we welcome extended comment that referees may be able to offer, we cannot impose on their good services beyond the expectation of an overall judgement on the article, together with brief justification of that judgement.

*What is the rate of acceptance/ rejection?*

The following statistics for 2008 and 2009 provide an indication of the pattern of acceptance/ non acceptance:

| Year | Accepted with no or minor revisions | Accepted after revisions | Not accepted |
|------|-------------------------------------|--------------------------|--------------|
| 2010 | 0                                   | 14                       | 42           |
| 2011 | 4                                   | 24                       | 58           |

Even an increase in the number of issues each year will not keep pace with the ever-increasing number of submissions. We can do little to mitigate the competition engendered by state funding policy and the kinds of incentive schemes that have become a feature of the higher education landscape.

*Is there an appeal mechanism should my article not be accepted?*

Beyond summarizing reasons for rejection – where applicable – we regret that we are unable to enter into detailed discussion on decisions reached by the Editorial Committee on the basis of referee reports.

*The journal describes itself as providing “a forum for scholarly understanding of the field of education”. What does this really mean?*

We understand this as implying that articles should represent a rigorous enquiry (conducted through argumentation or empirically) into the understanding of educational issues. Such inquiry originates in a problem rather than a solution, and it is rare for such enquiry to have no reference to, or engagement with, a broader literature and theory. Advocacy in the form of prescriptions or ‘how to do it’ recipe knowledge for practitioners seldom finds favour with referees. The question of audience is key. The assumed audience is the collective body of researchers rather than those more narrowly concerned with the effective implementation of specific policies.

Recent non-acceptances include a high proportion of undeveloped research reports, summaries of dissertations, and even sound but small-scale case studies that are purely context specific and unconnected with broader issues, literature or theory. Similarly, even a successful conference paper is usually in need of further development before it merits publication.