The teaching quality of mathematics lessons in South African schools

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Abstract

This study focuses on the quantification of the quality of mathematics teaching in 38 randomly selected sixth grade classrooms in the province of Gauteng, South Africa. The teaching quality is measured by coding videotaped lessons in three different components: mathematical proficiency, level of cognitive demand, and observed teacher knowledge. Results suggest that the majority of mathematics lessons in this province focus on procedural skills even when the intended lesson focused on conceptual understanding. In addition, most of the learners engaged only in low-level tasks and teachers demonstrated a lack of knowledge about how to integrate the content with effective pedagogical techniques.

Introduction

Despite widespread acceptance of the notion that improving learner performance may have a high economic and social payoff, policy analysts in all countries have surprisingly little hard data on which to base educational strategies for raising achievement. In South Africa this question is all the more pressing. South African learners score at low levels in mathematics and language tests even when compared with learners in other African countries (Plomp and Howie, 2006; Van der Berg and Louw, 2006). Further, the South African government's own evaluations of ten years of democracy show little improvement in educational outcomes despite significant policy changes (Department of Education (DoE), 2006). While some reasons for this poor performance may be evident, and there is widespread agreement that the main challenge in South Africa is the quality of education, there is little empirical analysis that helps policy makers understand the low level of learner performance in South African schools or how to improve it.

As a first step toward an empirical approach to unpacking the factors contributing to low levels of learning in South African schools, the Human Sciences Research Council in partnership with a consortium of South African universities and researchers at the School of Education at Stanford University engaged in a small scale empirical pilot study that focused on the role that teacher skills and practice play in South African learners' learning within the socio-economic and administrative conditions in those schools (and South African society more broadly). The team collected multiple forms of data, including learner, teacher, and school data (Carnoy, Gove and Marshall, 2007). This paper only focuses on the measure of teaching quality in mathematics lessons captured by video cameras. The purpose of this part of the study was to describe the teaching quality of the mathematic lessons. By 'teaching quality of mathematic lessons' we mean a composite of several aspects that characterise the teaching with the focus in the rigour and depth of the mathematics presented to the learners. These included the presence or absence of mathematical proficiency elements as intended by the curriculum or other teaching materials; the level of cognitive demand of the tasks the learners engaged in; and how efficiently the teacher uses her or his own mathematical and pedagogical knowledge to successfully implement the lesson. Even though there are other important aspects to consider when observing classroom lessons, we focus on the quality of the mathematics teaching because of its promising links with teacher knowledge and learner outcomes (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball, 2008; Kazima, Pillay and Adler, 2008; Hill, Rowan and Ball, 2005).

Imagine, for example, a Grade 6 classroom in which learners are learning how to identify two-dimensional shapes and three-dimensional objects. The teacher begins class by showing concrete models of objects such as cubes, pyramids, and objects around the classroom (such as learners' notebooks and bookshelves). The teacher then discussed the differences between a two-dimensional shape and a three-dimensional object and assigns a task. Learners must first cut different objects out of paper and then work in small groups to classify the objects according to dimensionality. After about thirty minutes of the teacher supervising learners' cutting and pasting, the lesson wraps up with each group presenting their classifications and with the teacher acknowledgement of the learners' successful work. There are several aspects of this lesson to notice: the teacher draws attention to the properties of shapes and objects with concrete objects that learners can observe, manipulate, explore, and relate to everyday life. Learners are actively engaged as they discuss properties such as length, width, and height and mathematical vocabulary ('2D', '3D', and 'dimension'). The lesson gives the opportunity for the learners to work collaboratively arguing about the classification of the objects rather than just sitting together working individually. The lesson presents geometric ideas in a creative way rather than giving the learners a list of two-dimensional shapes and three-dimensional objects from a textbook which have already been classified.

However, inspecting this lesson more closely, we notice that, despite its engaging and masterful pedagogical techniques, the *mathematical quality* of the lesson is lacking. When discussing the properties of a three-dimensional object the teacher gives learners a definition of a three-dimensional object – that it is an object with length, breadth and height. She proceeds to use concrete objects to demonstrate this definition, but uses only rectangular prisms. While this is correct for rectangular prisms, it is not true in general for all 3-dimensional objects. In the observed lesson, many learners come to the conclusion that a three-dimensional object is defined by an object that has three measures, length, width, and height. We saw this when the learners presented their work, since several of them identified cylinders and regular triangular prisms as two-dimensional figures because they only have two measures (height and circumference for the cylinder, and height and length of side of the base for the prism). This is evidence of an over-generalised definition of a three-dimensional object. The teacher never addressed this incorrect generalisation made by the learners and accepted all classifications of shapes that the learners presented as correct. Further, the resource materials that the teacher used in the lesson represented a deviation from a well-designed lesson presented in a textbook. That is, the *mathematical teaching quality* of the lesson was low, despite engaging instructional activities that fit many of the new South African curriculum requirements.

When analysing the lessons, we focus on aspects that determine the teaching as well as the quality of the mathematics. So, for example, while some observational instruments might have ranked the above lesson highly for its high-level cognitive task, engaging discourse, hands-on activities, and collaborative work, mathematics educator observers would have serious concerns over its content and consequential contribution to the development of learner misconceptions. The way we measure the lessons in this study is designed to capture these concerns.

Using a sample of 38 teachers from randomly selected schools in the Gauteng province of South Africa, we formulate the following questions:

- What is the level of attention to development of the strands of *mathematical proficiency* in the lesson?

- What is the *level of cognitive demand* in which the learners engage when the teacher implements the lesson?

- What is the *level of the observed mathematical and pedagogical knowledge* of the teacher during the lesson?

In what follows, we summarise the literature related to instruments designed to examine classroom practices in general and mathematics practices in particular. Next, we describe our method, including the use of our coding scheme by applying codes to specific lesson episodes. Finally, we present our results and discussion in the context of teacher education and professional development.

Measuring teaching quality in mathematics

Many research educators have observed school classrooms and measured their characteristics for different purposes (Shavelson, Webb and Burstein, 1986). The number of classroom observational instruments reviewed by Roshenshine and Furst (1973) and later Brophy and Good (1986) is almost as large as the number of studies reviewed – over 150. These particular instruments measured only behaviours related to teaching in general, such as pacing of instruction, classroom management, clarity and questioning the learners. More recently, in Latin-American countries, researchers have measured the time learners spend doing seat work, recitation activities, group work, and 'dead time' with the purpose of explaining the differences in learners outcomes across countries (Carnoy *et al.,* 2007). These aspects of teaching, although important, do not measure if the teacher is using the instructional techniques or behaviours in a way that is effective or consistent with the content goal of the lesson.

Perhaps in response to the need to explain better the mathematical aspects as well as the mathematical pedagogy harnessed during lessons, mathematics educators have turned their attention to the development of more specific observational protocols and instruments. What follows is a summary of the existing instruments and what constitutes the basis of our coding scheme in the context of South African schools.

Studying mathematics tasks and learner cognition during instruction

In our review of the literature, we found a set of instruments that focus on mathematical tasks and their implementation. One is the instrument used in the TIMSS video study (National Center for Education Statistics, 2003) to describe instructional practices in seven countries. Some of the aspects considered were 'Making connections', 'Stating concepts', 'Using procedures', and 'Giving results only', as intended by the mathematical content and as a result of the implementation of the lesson. Their major finding was that in Australia and the United States the lessons retained the 'Making connections' focus less often than the lessons in the other countries. A related instrument is presented in Henningsen and Stein (1997) where they investigate the factors associated with high-level mathematical tasks presented by the teacher and retained at high-level in the implementation by learners in the classroom. The high-level refers to one of the levels of *cognitive demands* defined by the authors as "the kind of thinking processes entailed in solving the task as announced by the teacher (during the set-up phase) and the thinking processes in which learners engage (during the implementation phase)" (p.529). The thinking processes they considered range from memorisation to complex thinking and reasoning. They found that the major factor that helps retain the high-level of cognitive demand is the effectiveness of the teacher in maintaining learners' engagement by scaffolding and consistently pressing them to provide meaningful explanations or make meaningful connections.

At more specific levels with respect to content, Gearhart, Saxe, Seltzer, Schlackman, Ching, Nasir, Fall, Bennett, Rhine and Sloan (1999) developed an instrument to code videotapes and field notes from 21 primary classrooms. Their purpose was to measure the effect of curriculum and professional development (opportunity-to-learn construct), in the context of teaching fractions, on learner achievement. Their instrument included aspects like "the degree to which practices elicit and build upon student thinking, the extent to which conceptual issues are addressed in treatments of problem solving, and the extent of opportunity to utilise and interpret representations in ways that help students build understandings of underlying mathematical concepts" (p.292). The last two proved to be significant aspects associated with learners' performance.

Collectively, these studies suggest that when studying the quality of mathematics instruction, it is important to include aspects that describe the mathematics the learners have the opportunity to learn intended by the curriculum or other instructional materials as well as the way the mathematics is presented and assimilated by the learners. In other words, we need to focus on the *mathematics* that is available to the learner, independently of the way the teacher is teaching it and the learners are learning it; but also on what the *learners* get out of a mathematics lesson measured by the level of cognitive demands. In the international comparison literature, Carnoy *et al*. (2007) investigate these two domains in lessons across three Latin America countries (Cuba, Chile and Brazil) to explain the differences on learner outcomes in those countries. The authors used as a framework to describe the mathematics available to the learner, the five strands of mathematical proficiency that the National Research Council (2001) publication *Adding It Up* sets out as necessary for anyone to learn mathematics successfully and which they define as mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These strands have some similarities with the TIMSS framework listed above and with the United States' National Assessment of Educational Progress (NAEP) framework. To describe the thinking process the learners engaged during instruction, Carnoy *et al*. (2007) use the levels of cognitive demand defined by Henningsen and Stein (1997) and Stein, Smith, Henningsen and Silver (2000). They found that learners in Cuba, which had the highest learner outcomes in relation to the other countries, engaged in higher levels of cognitive demand and their curriculum also gave more opportunities for the development of mathematical proficiency (i.e. more representation of all five strands). What this study and the ones mentioned above do not take into account is the third key player during instruction – the teacher. Fortunately, there is another body of research that has focused on teachers' knowledge and skills during instruction; we summarise it in the next section.

Studying teachers' knowledge during instruction

Measuring the way teachers apply what they know (mathematically and pedagogically) to teach effectively has been studied more recently. Hill and colleagues (2008) give an extensive review of literature in this area and provide an instrument that captures aspects focusing on the mathematical quality of instruction (MQI instrument) and its relation to their measures for mathematical knowledge for teaching (MKT). The aspects that this instrument measures are mainly based on the theoretical and empirical work defining the MKT construct (Ball and Bass, 2000; Ball, Hill and Bass, 2005; Hill, Shilling and Ball, 2004). The MQI instrument measures aspects that focus on the teachers' skills and knowledge during instruction. These include mathematical errors, responding to learners inappropriately or appropriately in terms of the mathematics, connecting practices to mathematics, richness of the mathematics, and mathematical language. In this exploratory study, the authors found a significant association between levels of MKT and the mathematical quality of instruction. Previously, Hill *et al.* (2005) found a positive association between MKT and learner achievement gains, which supports the contention that the teachers' mathematical knowledge during instruction is ultimately related to learner achievement. In South Africa, Kazima *et al.* (2008) and Adler and Pillay (2007) have used case studies that build on the work of Deborah Ball and Heather Hill to provide more detailed insights. For example, they argue that the mathematical work that teachers do needs "to be understood. . . in relation to particular topics in mathematics, and to particular approaches to teaching". (Kazima *et al*., p. 296). Therefore, when measuring teachers' mathematical work during instruction, we also need to take into consideration the topic or main goal of the lesson and the particular teaching approach, which is related to pedagogical perspectives.

Taking all these aspects into consideration, we design a new instrument that will not only help describe the teaching quality of the mathematics but, in the future, also can be linked to teacher and learner outcomes. In the next section, we describe in more detail the aspects of our instrument, sample and data collection, and analysis.

Method

The quantitative method to describe the teaching quality of mathematics was driven by the main goal of the larger project (Carnoy *et al*., 2007) of which this study is a part. This was to unpack the factors contributing to low levels of learning in South African schools by focusing on the role that the teacher skills and practices play in learners' learning within the socioeconomic and administrative conditions in those schools. More specifically, results from the larger study seem to suggest that there is a relationship between the teaching quality and teachers' knowledge in a paper-and-pencil assessment, similar to the work of Hill *et al*., (2008) and Marshall and Sorto (2011). In this paper, we only describe the development of the instrument used to code the lessons and report on the descriptive part of the teaching quality since the original project was a pilot study and the quantitative relationships found were more exploratory in nature.

Sampling and data collection

A random sample of 40 schools was drawn from the Department of Education records in Gauteng Province. The sample schools were spread across five district municipalities, the City of Johannesburg, City of Tshwane, Ekurhuleni, West Rand, and Sedibeng. These district municipalities cut across a diverse area of formerly racially segregated, rich and poor neighborhoods and schools. One Muslim school was not videotaped because of religious considerations and in one school an English lesson was videotaped instead of a mathematics lesson. Thus, the lesson analyses are for 38 sixth-grade mathematics lessons in 38 schools. Although the teacher sample is probably not completely representative of the province, it provides us with a good set of data to describe the teaching quality of mathematics lessons distributed across different categories of schools.

The filming was done towards the end of the school year (one lesson for each teacher) by previously trained personnel of the South African team. Teachers were notified in advance about the research team visits and all of the lessons observed were about teaching mathematical content as opposed to review sessions for upcoming assessments.

Framework and instrument development

The framework to characterise the teaching quality of the lessons for this study is a product of several sources. These include our experience as mathematics teachers and mathematics teacher educators; our experience studying teaching in developing countries in Latin America; our experience working with teachers in the Gauteng province; and the existing literature that investigates mathematics instruction. The development of the codes started with observational classroom codes used in rural Guatemala (Marshall, 2003 and Marshall and Sorto, 2011). From this study, we learned that mathematics instruction in rural settings was much less complex in terms of pedagogical techniques and use of resources than most lessons studied in more developed countries like United States, Germany, and Japan. The limitation of this study is that the codes did not include the measurement of the level of content or curriculum that was being covered. For the next study analysing 42 videotaped lessons in Brazil, Chile, and Cuba (Carnoy *et al.*, 2007) the framework used in Guatemala was extended to include codes that addressed not just general pedagogical processes but level of mathematical proficiency and levels of cognitive demand. The purpose of this analysis was to explain differences in academic achievement among the three countries that could not be explained by statistical methods such as education production functions and hierarchical linear modeling (Carnoy and Marshall, 2008). Even though the addition of new codes helped sharpen the focus on the intended and implemented mathematical tasks, the framework was missing that which other authors (Adler and Pillay, 2007, Hill *et al*., 2008) argue is needed when analysing the quality of mathematical instruction. In response to these kinds of concerns, we included one more aspect – the *level of the observed teachers' knowledge while teaching*. These new codes were tested for the first time in the Panama and Costa Rica study (Sorto *et al*., 2009) with 50-videotaped lessons.

We now turn to a detailed description of the three major components of the framework used in South African schools: the mathematical proficiency the learners have the opportunity to acquire; the level of cognitive demand the learners are engaged in during the lesson; and the observed teacher's knowledge while teaching.

Mathematical proficiency. This is measured by evidence of the development of any of the five strands that form the mathematical proficiency variable, which according to *Adding It Up* (National Research Council, 2001) are necessary to learn mathematics successfully. The five strands are:

- conceptual understanding – comprehension of mathematical concepts, operations and relations;
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- *strategic competence* – ability to formulate, represent, and solve mathematical problems;
- *adaptive reasoning* – capacity for logical thought, reflection, explanation and justification; and
- *productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p.116).

These strands are not taken as individual goals but rather as interdependent and interwoven aspects required for the development of mathematical proficiency. If any one of the five elements is missing, the learning process is not considered complete. Nevertheless, in the context of evaluating a (short) lesson it may be unrealistic to expect development of all five elements – even in a very good class. This calls for some flexibility in how we assess the mathematical proficiency of the lesson. The focus in this component is the *mathematics* available to the learner.

Cognitive demand. The level(s) of cognitive demand in which learners engage during the lesson are derived from a rubric in Stein *et al*. (2000) classification of higher and lower cognitive demand. These include:

- *Memorisation* – recollection of facts, formulae, or definitions.
- *Procedures without connections* – performing algorithmic type of problems and having no connection to the underlying concept or meaning.
- *Procedures with connections* – use of procedures with the purpose of developing deeper levels of understanding concepts or ideas.
- *Doing mathematics* – complex and nonalgorithmic thinking, learners explore and investigate the nature of the concepts and relationships.

The focus in this component is the thinking processes in which *learners* engage.

Observed teacher's knowledge. We characterise the *observed* teachers' knowledge in a lesson by focusing on three aspects. The work of Shulman (1986) forms the basis of these categories. These include:

- *Grade level mathematics knowledge* – the presence of computational, linguistic, and representational accuracy for the mathematics at the grade level being taught. We take into account any mathematical errors during instruction.
- *General pedagogical knowledge* – the use of general instructional techniques beyond the lecture mode. Elements include how well the teacher has all the learners engaged, his/her use of proper classroom management techniques and the quality of instructional materials.
- *Mathematical knowledge in teaching* – the degree to which teacher can appropriately integrate the use of the instructional techniques with the mathematical concept being taught and its effectiveness on learning.

This also includes the use of correct language to clearly convey mathematical ideas clearly.

Together these three analytical elements make it possible to go beyond a simple reconstruction of each lesson and consider the deeper mathematical meaning of what is being taught. These elements also allow us to assess what the teachers know and how they apply this knowledge in the classroom.This in turn makes for some useful linkages between the lesson analysis and teacher questionnaires.

Of course, what is observed in one lesson does not measure the entire body of knowledge a teacher has in mathematics, or any of the other kinds of knowledge. The purpose of looking at the teacher's knowledge for these lessons is not to characterise the entire knowledge of a teacher. For this we would need a case study where we observe a teacher for a long period of time. The purpose is to measure how well the teacher uses these specific knowledge forms in a particular lesson.

Coding and inter-rater reliability

To capture the presence of the twelve different components (five components for mathematical proficiency, four components for cognitive demand, and three for observed teacher's knowledge) a coding system was used for each lesson. After observing a particular lesson the researcher adjudicated a code of 'present' (P) or 'not present' (NP) for each component that defines the three elements of teaching mentioned above. A conservative judgment was used for the 'present' code. That is, if the component was observed at least once during the lesson, a code of P was adjudicated. Other video studies (e.g. Hill *et al.*, 2008, Andrews, 2009) have broken the lessons into small segments of five or ten minutes or episodes to account for the complexity of instruction. However when this method was applied to our lessons, we did not find significant differences between the codes considering the lesson as a whole compared to and lessons broken into smaller segments. For logistical reasons we did not conduct any type of structured or semi-structured interviews with the teachers who were videotaped. This is a limitation because interviews would have allowed us not just to validate our coding system but also to enrich our understanding of teaching practices in these countries. In addition, an overall evaluation of the teaching quality observed in the lesson was assigned using the scale 1 (*low*), 2 (*medium*), or 3 (*high*). These ratings were a holistic composite of the 12 components described above.

We (two researchers) first worked independently to code each lesson, and then reconciled our codes by holding discussions about any disagreements. We agreed most of the time and there were only few instances where we needed to discuss discrepancies in coding. Inter-rater reliability between the two of us ranged between 85 per cent to 100 per cent for individual codes.

Results

A result that stands out is the large class sizes in Gauteng, on average 37.1 learners in our sample, varying from 11 to 67 (standard deviation equal to 10.1). The length of classes we observed were almost all between 30 to 40 minutes long.

With respect to what teachers and learners do in the classroom, we characterise a typical mathematics lesson in Gauteng's sixth grade classrooms. About one-third of the lesson time is teacher-led, in which the teacher is presenting the content to the whole class. Another one-third of the lesson time is taken by the teacher asking questions to the class which are answered by individual learners or in chorus. (On average, for the 38 classrooms we timed, 44 per cent of the recitation time was individual responses, 36 per cent was chorus, 15 per cent was solving at the blackboard, and 5 per cent was groups reporting). The final third was seat work. Much of the recitation time (individual learners and learner chorus responding to the teacher) is mixed in with teacher-led talking about mathematical content. In the highest SES schools, more time is spent on whole class teacher presentations and on seat work, and less on recitation. In the lower SES classrooms, learners are more likely to be seated with their desks grouped into 4–6 learners facing each other, although when the learners in such grouped situations are doing seat work, it is almost entirely individual. Actual work in groups constitutes only about 4 per cent of class time.

We now turn our attention to an analysis of the teaching quality of the content. The mathematics content observed in the entire set of lessons was evenly distributed between three major mathematical areas: number concepts and operations, geometry, and measurement. We only observed one lesson in the area of data handling and probability and one lesson in the area of Patterns, functions, and algebra. Table 1 summarises the results according to the three major elements of our quality of teaching construct.

Table 1: Percentage of lessons in which each component was observed

	Per cent of lessons
Mathematical proficiency:	
Conceptual understanding	46.15
Procedural fluency	76.92
Strategic competency	12.82
Adaptive reasoning	46.15
Productive disposition	33.33
Cognitive demand:	
Memorisation	74.36
Procedures without connections	58.97
Procedures with connections	41.03
Doing mathematics	5.13
Observed teachers' knowledge:	
Grade level mathematics knowledge	97.44
General pedagogical knowledge	92.31
Mathematical knowledge in teaching	25.64
Total lessons	38

Mathematical proficiency

It is a lot to expect opportunities for the development of all five of the mathematical proficiency strands to be present in individual lessons, especially in short lessons. Instead we were more concerned about the extent to which all strands turn up in the overall summary of multiple lessons. In other words, are there specific elements of proficiency that are largely absent from these classrooms as a whole?

The overall pattern of the development of proficiency in mathematics in South Africa is somehow balanced with respect to some of the components (see Table 2). Even though the majority of the lessons provide for procedural aspects of mathematics learning, nearly half of them also provide for the conceptual and reasoning aspects. It was clear from the observations that some teachers value conceptual understanding before learners move to the manipulation of symbols or computation. This is also consistent with the kinds of questions used by teachers in the classroom. However, as we will see later, not all teachers were able to teach conceptually in an efficient way. There were few instances where learners had to show the ability to formulate, represent, and solve mathematical problems, also known as strategic competence. When this strand was observed the learners were given mathematical problems applied to real word situations and asked to apply their knowledge of previous mathematics content learned to arrive at a solution. There were a few examples where development of strategic competence was observed. In all of these observed lessons learners were either engaged in whole class or group discussions. In one lesson where the mathematics focused on the concept of fractions, learners were solving problems, making conjectures, and sharing their reasoning in relation to questions involving dividing sausages and groups of apples into different fraction parts. The teacher demonstrated excellent questioning and guiding skills. In another lesson the learners were actively involved in making models of 3-D shapes from which they could then count the faces, edges and vertices in order to analyse and compare 3-D shapes. A third example, on the topic of time (Measurement), the teacher assigned questions to groups and gave time for them to work out their solutions before they had to present these solutions to the class. When learners presented, he pushed them to explain their answers and enabled them to understand where they had gone wrong. Learners had to rethink when necessary and had to give clear explanations for their solutions.

For the lessons that lack the aspects of ‘Reasoning’, rules, definitions, and procedures were often presented without the teacher providing an opportunity for learners to consider why they were true. When learners were involved in working on a problem or asked to give an answer, they were not expected to explain their reasoning or provide a valid justification. Many educators refer to this type of teaching as ‘answer-centred’. In one, where the teacher went over a fairly traditional worksheet that learners had evidently done as homework. The sheet called for writing numbers in words, giving values of underlined digits in several five digit numbers, writing numbers represented on abacus diagrams and writing numerals for numbers given in words. The class chorused when called to do so and individual learners wrote their solutions on the board, once the solutions had been confirmed by the teacher. There was no discussion and there were no questions that created opportunities for reasoning. An extreme example was a lesson where the learners spent the whole time copying down information from the board onto a chart. The teacher circulated answering questions very curtly. She seemed to just want them to get on with the copying. The words ‘copy’ and ‘copied’ very often formed part of her answers.

Finally, the productive disposition strand refers to learners seeing mathematics as sensible, useful, and worthwhile combined with a belief in their ability to do the maths. This category was observed only during the lessons where learners were either involved in the application or reasoning of mathematics. This occurred in about half the lessons. However, in those lessons learners seemed to enjoy and value the logical thinking and problem solving activities.

Level of cognitive demand

Beyond the topic covered in the lesson, the kind and level of thinking required of learners on a particular topic or mathematical task impacts on the quality of the learning experience. The measure of the level of cognitive demand enriches and relates to our previous measurement of mathematical proficiency. A large percentage of the lessons (74.36 per cent) required learners to simply recall rules and definitions or perform algorithms with no relation to the underlying concepts. Opposite patterns are observed for the higher-level cognitive demands. A smaller percentage of lessons require learners to understand the meaning of operations or underlying concepts behind the procedures and a very small per cent require learners to investigate or explore relationship between mathematical ideas. The distribution of lessons for the first three levels is to some degree uniform.

We have an important observation about the level of cognitive demand based on the lessons we saw in South Africa. The observed level was the one *implemented* by the teacher and not necessarily the level *intended*. For example, the videotaped lessons show that about 50 per cent of the teachers had intended to deliver a higher-level lesson, guided by textbooks, pre-prepared activities, and concrete models. However, only about 26 per cent successfully implemented such lessons. The South African lesson designs tend to include demanding questions, but the actual formulation and sequence of questions does not always make it possible to probe the learners' conceptual understanding. These findings are consistent with results from the TIMSS 1999 Video Study and with findings by Stein *et al*. Mathematical tasks or problems with high level cognitive demands "are most difficult to implement well, frequently being transformed into less-demanding tasks during instruction" (2000, p. 4).

Another important observation was the lack of coherence in a large percentage of lessons. Teachers tend not to have a clear goal of the lesson. Some of the lessons started with a short mini lesson on some topic and ended with an 'activity' related to the topic, but unrelated to the mini lesson. Often the teacher does a mini lesson and then does not follow up with other activities. This is a big problem – lessons do not have sufficient substance to allow learners opportunities to consolidate what has been learned. The other problematic pattern observed was the lack of whole class discussion on the activities or worksheets. The 'discussion' is often just a chorus of agreement to given answers – or the completion of comments-prompted answers. These really give no indication as to whether or not the learner actually was able to give the answer him/herself.

The teacher's observed knowledge

In this part of the analysis we turn to observations to classify teacher knowledge. This is a novel approach with few antecedents (Hill *et al*., 2008; Sorto *et al*., 2009), and implementing it presents a number of challenges. It clearly requires mathematics education experts to classify the teacher's knowledge based on his/her actions and choices in the classroom.

For content knowledge there are a number of possible 'clues' for assessing what the teacher knows. It is fairly straightforward to focus on the examples of problems they solve in class or the corrections they make of learner mistakes. Careless mistakes when teaching operations or procedures, or more serious misconceptions about underlying concepts, are each indicators of content knowledge deficiencies. This same standard can also be applied to higher level content knowledge, although we expect this element to be less applicable in the average lesson.

There are also the general pedagogical skills we referred to earlier, although we have not compiled a complete list of these actions. Once again a trained expert in the subject with extensive experience observing teachers is needed to classify the teacher's *pedagogical knowledge*. Elements include how well the teacher has all of the learners engaged, his/her use of proper classroom management techniques, and the quality of instructional materials.

The third and final domain of knowledge is formed by the integration of the two previous knowledge areas. This *mathematics knowledge in teaching* is not necessarily separate knowledge, but it is demonstrated in the class by how well a teacher uses mathematical and pedagogical knowledge to help learners learn mathematics.

Table 1 shows the percentage of teachers that demonstrated knowledge in each of the kinds of knowledge described above. One important note is that the kind of knowledge demonstrated was connected with the goal and level of cognitive demand of the lesson.

For the mathematical knowledge category, teachers were coded according to demonstrated knowledge of the mathematics by the correctness in their written and spoken mathematical statements. Table 2 shows a description of some of these errors or incorrect statements and their significance in terms of the teaching and learning of the content. Most of these errors were related to the inappropriate use of the terminology and lack of accuracy in the mathematical language when explaining the concept. Most of these incorrect statements or inappropriate explanations were coded as lack of mathematical knowledge in teaching.

Table 2: Errors of expression, concepts incorrectly explained by teachers

Errors observed	Mathematical concept involved	Significance
1	Wrote 9+(64-8)=(9+56) x 64.	Careless error
2	When finding ⅓ of 30, the teacher writes out the numeric algorithm, calls it a 'proof' of the value of ⅓ of 30.	Inappropriate use of mathematical language and significant in the teaching of fractions.
3	Says a bucket is a 2-D shape since it has no sides.	Confusion with properties and identification of 2-D shapes and 3-D objects.
4	Position of angles incorrectly shown on the 360° 'standard positions' in rotation. Says that a reflex angle is from 180° degrees to 270°. Draws a 90° angle as a semi-circle.	Confusion of geometric concepts and terminology relating to angle.
5	Says that the opposite angles of a parallelogram are not equal.	Just careless in this instance.
6	The concept of a point is explained, using the example of the sun as the origin of a ray. But teacher confuses learners who say the 'sun cannot be a point'.	Conceptual explanation – visualisation of geometric concepts leads to confusing explanations.
7	Draws a line and a line segment on the board, but labels them incorrectly and speaks about them incorrectly – calls line a line segment and vice versa.	Confusion of terminology/ naming of geometric shapes.
8	Fraction terminology – calls a mixed number a mixed fraction. Says simplest form 'I must always convert it back to a mixed fraction' (e.g. 28/5 is not in simplest form).	Terminology and concept – fractions in simplest form; mixed numbers.
9	When speaking about fractions, reads 1 2/4 as 'one over two four' throughout the lesson.	Inappropriate use of mathematical language. Reading fraction numerals as words.
10	Calls a protractor a ruler throughout lesson.	Terminology
11	Says that place value and total value are the same thing.	Place value terminology affecting conceptual understanding.

In terms of pedagogical knowledge, there was evidence of the knowledge of pedagogical techniques. In particular, the use of concrete models to illustrate concepts and the more frequent use of hands-on activities such as cutting, colouring and pasting. This measure is linked with the intended level of cognitive demand of the lesson analysed above. The final element is the degree of effectiveness of the use of these techniques and how well they were connected with the mathematical concept being taught. Note the small per cent of teachers (26 per cent) in this category. Some teachers in this category showed a well-planned lesson with a rich task presented to learners and a good ‘flow’ of the lesson. Others were effective because of the powerful explanations and skilful level of communication on the part of the teacher to bring the complex mathematical ideas to the level of the learner. The better teachers used questioning to elicit answers given independently by learners, from which an observer can say that the learner has understood what he/she is talking about.

Conclusion

There is a large variation in terms of teaching quality levels in the Gauteng province of South Africa. We did observe classs with high levels of opportunity for the development of mathematical proficiency where learners where engaged in high-level cognitive tasks, engaging discourse, hands-on activities, collaborative work, and teachers that demonstrated skills and knowledge of mathematics and pedagogy. Sadly, these classs were the exception.

A typical mathematics lesson in the province of Gauteng, South Africa is characterised by a teacher lecturing about a concept or a topic for a short time, doing an example of an exercise on the board, and then the learners work in their notebooks doing similar problems for the rest of the time. Characteristically few (two to six) problems are set for the learners. The teaching focuses mainly on procedural skills and the learners are engaged in cognitively low-level tasks. Teachers demonstrate knowledge of the mathematical content at the grade they are teaching and also demonstrate knowledge of general pedagogical techniques. However, most teachers do not integrate these two domains of knowledge effectively. More specifically, most teachers do not have a clear aim or goal for their lessons and they do not present the learners with a well-sequenced series of activities that help the learners acquire the underlying mathematical concept. Further, many of them do not use proper mathematical language when trying to explain the concepts and they lack ability to effectively use models and multiple representations to illustrate abstract concepts. This evaluation of the quality of mathematics teaching as evidenced in a sample of 38 schools and its implications will be further investigated in the full comparative study.

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