Specialising pedagogic text and time in Foundation Phase numeracy classrooms

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Abstract

In this paper we focus on an aspect of the crisis we face in mathematics teaching and learning in South Africa at the present time, namely the teaching of number in the Foundation Phase. We analyse classroom observation data collected in 18 classrooms across Grades 1 to 3 in three different schools serving very poor communities and use the notion of semantic density to highlight the degree of specialisation of content and mode of representation across lesson time. The central findings of the paper are that while we can discern a trajectory of development from counting to more abstract ways of working with number across Grade 1 to Grade 3, students remain highly dependent on concrete strategies for solving problems at Grade 3 level. Learners' opportunities to grasp the symbolic system of mathematics are inhibited by classroom practices that privilege concrete modes of representation, which restrict access to more abstract ways of working with number, and by the inefficient use of class time.

Introduction

Describing the 'brute inequity' of primary school achievement in South Africa, Brahm Fleisch (2008) assembles data (produced by government agencies and others) which spells out graphically and unambiguously the

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debacle we face in literacy and mathematics achievement in primary schools in this country. He shows that at the end of 2001, for example, testing of a large sample of Grade 3 learners revealed very poor performance in elementary mathematics – learners achieved an average score of 30 per cent on the numeracy tasks set. By 2007 this had increased to 35 per cent. Grade 6 achievement figures released by the Department of Education in 2005 (Department of Education, 2005) showed that 81 per cent of learners were not achieving at the levels specified by the National Curriculum Statement. As documentation produced by WCED indicates, Grade 3 mathematics results in 2006 showed that more than 60 per cent of Grade 3 learners were performing below the expected level for literacy and numeracy. It also revealed that between 2002 and 2006, numeracy levels dropped from 36.6 per cent in 2002 to 32 per cent in 2006. (Western Cape Education Department, 2006). Eric Schollar (2008) makes the sobering point that only 1.5 per cent of the 1995 Grade 1 cohort achieved higher grade passes in mathematics in the 2006 matriculation examination. Fleisch (2008) goes on to illustrate the degree of South Africa's underperformance by citing regional and international studies which place South Africa lower in numeracy achievement than eleven other African countries, including Madagascar, Malawi, Zambia and Botswana.

The distribution of success and failure in primary mathematics mirrors the differentiation of schooling according to the social class background of learners, producing a consistent bimodal outcome. A number of studies have attempted to provide explanations of this pattern of achievement. Hoadley's (2007) study for example describes the different kinds of knowledge made available to primary school learners from different social class backgrounds. While working class children in her study were exposed to localised and everyday knowledge, middle class children were given greater exposure to the specialised knowledge required for success in school. Fleisch (2008) cites Reeves (2006) as attributing performance at Grade 6 level to problems with curriculum coverage, coherence, cognitive demand and pacing.

In this paper we focus on one aspect of this crisis in performance, namely the teaching of number in the Foundation Phase. We identify a set of problems which we believe become compounded over the years and which hold learners back from achieving in mathematics higher up in the education system. In setting out our conclusions we do not depart substantially from the findings of Hoadley, Reeves and others, but contextualise the problem very specifically in terms of the teaching of number in the early years of schooling.

The research project from which our data is taken was designed to explore the impact of a professional development programme on teaching numeracy in the Foundation Phase. Classrooms were observed in 2004 and 2006 and we have analysed the shifts in pedagogic practices that have resulted from the professional development programme. However, in this present paper we are not directly concerned with the relationship between professional education and classroom teaching, and the shifts in teachers' practices over time, but rather with key common features of pedagogic practice in the Foundation years taken as a whole. Here we present an analysis of classroom observation data which was one of three data sets, including also interviews with teachers and learner achievement scores in Grades 1, 2 and 3 over three years.

We video-taped nine teachers in two years – 2004 and 2006 – in one lesson in each year, to generate 18 recorded lessons. We recorded only one lesson on each occasion and did not spend sustained periods of time in the schools. On the basis of such restricted access to classrooms we would not normally make strong claims about the extent to which the lessons can be regarded as representative of teaching as usual in the schools. However, we were struck by the uniformity of practice across nine teachers in all three schools, across time, and it is this commonality of practice that forms the basis of our analysis. Although some changes in pedagogic practice were evident between 2004 and 2006, most especially around the use of apparatus, the organisation of lessons remained substantially the same over time.

A number of researchers have been involved in the analysis of the classroom data discussed here. We worked as a team drawing out common categories for analysis which we then refined iteratively over time. A broad range of crucial issues emerged – the relationship between teacher talk and learner talk, the ways in which teachers 'scaffold' learning, the relationship between whole group teaching and individual effort by students, between verbal and written work, between differentiated and undifferentiated tasks for different learners and the whole matter of control by teachers and learners respectively over selection, sequencing and pacing of tasks (Schmitt, 2009; Jacklin and Hardman, 2008). These aspects are important, and will become the focus of a series of future papers. The present paper concentrates on a particular aspect of teaching mathematics – the shift from concrete to symbolic reasoning. We are interested in how children learn to operate with ever more abstract and sophisticated representations of number, and how teachers assist them in doing this. We have distilled the concerns of our study in the following terms: how do teachers, through the construction of tasks, specialise pedagogic text and time in the teaching of number? In other words, how do they move from the concrete, local experience to engaging with more abstract forms of knowledge?

The central findings of the paper are that while we can discern a trajectory of development across Grade 1 to Grade 3 from counting to more abstract ways of working with number, students remain highly dependent on concrete strategies for solving problems at Grade 3 level. Recent classroom-based research in mathematics shows the dominance of concrete methods such as tally counting in solving problems in mathematics in the early grades (Kühne, 2004; Hoadley, 2007; Schollar, 2008). The inefficiency of such methods, and the failure of many students to abstract from concrete representations, has been offered as a significant contributor to poor mathematics achievement of students in South African schools. We argue that learners' opportunities to grasp the symbolic system of mathematics is limited by classroom practices that privilege concrete modes of representation, which inhibit access to more abstract ways of working with number, and by inefficient use of class time.

Counting, calculating and arithmetic

We expect that by the time they leave primary school, children will have a confident grasp of counting, number and arithmetic, which will provide a solid platform for them to engage with algebra and other aspects of the school mathematics curriculum when they reach secondary school. Evidence suggests that the majority of children do not achieve this competency at the end of primary school, and a deficit emerges in the early years that becomes augmented when students reach high school. There are three interlinked aspects to mastering Foundation Phase numeracy: progression in acquiring the number concept, the shift from concrete to abstract reasoning, and relatedly, the move from counting to calculating.

Mastery of counting

A progression is developed below which suggests the pathway that children need to tread in order to gain mastery of arithmetic. It is not presented as a strict hierarchy (either here or in the literature[2] from which it draws) although there is a strong trajectory underpinning it.

Gelman and Gallistel (1986) suggest that children can be deemed to have mastered counting[3] when the following principles are in place:

- *The 1-1 principle*. This entails children being able to mark off items in a collection with distinct markers or tags so that one and only one marker is used for each item. The child has to learn to co-ordinate two processes – partitioning (differentiating within the collection between items which have been counted, and those which have yet to be counted, which may be achieved physically or mentally) and tagging or marking (drawing on distinct markers or tags, one at a time.) The two processes have to work simultaneously, beginning and ending at the same time. Three kinds of errors can arise in this – errors in partitioning, such as tagging an item more than once, errors in the use of tags (such as using one more than once), and failure to co-ordinate the two processes adequately.

- *The stable order principle*. Counting entails more than assigning arbitrary markers to items in a collection. Children also need to recognise that the tags themselves are organised in a repeatable, stable order. Gelman and Gallistel comment that the human mind "has great difficulty in forming long, stably recallable lists of arbitrary names (words)" (1986, p.79). They argue that much of a child's first engagement with learning number is rote learning the first 12 or 13 number words, and the rules that generate subsequent words. Fuson and

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[2] It is deeply problematic that we have so little large-scale research in South Africa (apart from large-scale testing) that can inform us about children's capabilities in number in the early years. A significant corpus exists in Britain, Europe and the USA on number acquisition from birth to well into the primary years, and in the absence of locally-based research we have been obliged to draw on this corpus to understand what is going on, and going wrong, in the early years of education in South Africa.

[3] Dehaene (1992) suggests that the set of requirements generated by Gelman and Gallistel places strong demands on the identification of counting in children. He sets out two opposing theoretical positions: that of Gelman and Gallistel, who advance what he refers to as the "principles-first theory", arguing that the "principles are innate and guide the acquisition of counting procedures" (p.11); and the "principles-after" theory, advanced by Fuson and others, who suggest that "counting principles are progressively abstracted, in a Piagetian manner, after repeated practice with imitation-derived rote counting procedures" (p.11). See also Nunes and Bryant (1996).

Kwon (1992) point to the particular difficulties learners encounter in learning to count in English (in contrast, for example, to Chinese and Japanese learners) which extends to difficulties with grasping the notion of place value.

- *The cardinal principle*. This entails understanding that a number, such as five, can be achieved by counting the items of a set of five objects, and that it represents the total number of items in a set. This understanding is crucial to all of the child's later number reasoning – that a number such as 5 encapsulates numerosity (counting items 1 through to 5) but also that 5 represents the total number of items, and becomes an object which can be manipulated.
- *The abstraction principle*. This is the understanding that counting procedures can be applied to any collection of items. Steffe, Von Glasersfeld, Richards and Cobb (1983) argue that there are five different types of countable item, progressively difficult for the child to manage: perceptual units (which can be seen), figural items (items not present, but recallable – for example the number of dwellers in a home), motor units (movements like steps or handclaps), verbal units (utterances of number words), and abstract units.
- *The order irrelevance principle*, which entails an understanding that the order in which items in a collection are counted does not affect its numerosity. This entails an understanding that the tag applied to an item is arbitrary and is not a characteristic of the item, and that the same cardinal number applies to a collection irrespective of the order in which the items are counted.

Once these principles have been mastered (and this usually happens over a protracted period), children have developed the ability to work with numbers as representations of numerosity. As Gelman and Gallistel put it: "counting provides the representations of reality upon which the [numerical] reasoning principles operate. That is, counting serves to connect a set of reasoning principles to reality" (1986, p.161).

From process to concept

A significant leap in understanding is entailed as a child moves away from regarding numbers as reflecting numerosities, to objects which can be manipulated according to certain laws. They understand that the counting numbers do reflect numerosity, but that numbers such as -5, root 2, or pi, do not. As Gray (2008) comments "It [the formation of numerical concepts] involves a shift in attention from the objects of the real world to objects of the arithmetical world – numbers and their symbols" (p.82). "Numerical symbols do not represent either a process or an object: they represent both at the same time" (p.88). He and Tall give this compression the name *procept*.

Gray and Tall (2007) suggest that there are three distinct types of mathematical concept: "one based on the perception of objects, a second based on processes that are symbolised and conceived dually as process or object (procept) and a third based on a list of properties that acts as a concept definition for the construction of axiomatic systems in advanced mathematical thinking" (p.23). They refer to the "*proceptual divide* between those who cling to the comfort of counting procedures that, at best, enable them to solve simple problems by counting and those who develop a more flexible form of arithmetic in which the symbols can be used dually as processes or as concepts to manipulate mentally. *Proceptual thinking* occurs when counting procedures are compressed into number concepts with rich connections – for example, knowing things like '4 and 2 makes 6, so 6 take away 4 must be 2' and using these 'things' to derive new knowledge such as '26 take away 4 must be 22 because 26 is just 20 and 6'" (p.26–29). Authors such as Piaget, Skemp, Fischbein, Bruner, Biggs and Cillis, drive at the distinction between the concrete and the abstract in different ways, argue Gray and Tall, and in ways which align with the 'three worlds of mathematics' they refer to: the conceptual-embodied, the proceptual-symbolic and the axiomatic-formal.

Until learners can make the shift from process to concept, they will not be able to understand that 10 is a concept, and will not be able to comprehend two digit numbers, and place value. The same applies to sharing and producing fractions, which must then be understood as numbers which can be manipulated. "If fractions are seen as procedures, then addition is almost too complicated to contemplate" (Gray and Tall, 2007, p.33).

From counting to calculating

The shift from counting, to calculation which makes use of counting strategies, to calculation which does not rely on counting, takes place across Grades 1 to 3, and beyond. As Gray, Pitta and Tall (2000) comment, as learners move from the perceptual world (the use of counters, fingers and so forth) to using representations of it (the use of tallies, and later number words) they are engaged in processes which are basically similar in that all are analogous to the process of counting. “The concept of unit becomes wholly abstract when the child no longer needs any material to create countable items nor is it necessary to use any counting process” (p.403).

Gelman and Gallistel underscore this move from counting, to calculating strategies based on this, and finally to calculation which does not rely on counting, by reflecting on the difference between what they term numerical and arithmetic reasoning. Children’s early numerical reasoning relies on the numerosities of sets which are produced through the counting process. A number, in terms of this reasoning, derives from the process of counting. Arithmetic reasoning, in contrast, grasps that “the laws of arithmetic govern an abstraction called number” which has no reference to numerosity. It is the laws of arithmetic that determine what is and is not a number, and not the reverse (p.181).

This emerging understanding of number, and of arithmetic, requires that teachers assist learners to deepen notions of counting, develop flexible and powerful means of representing number using apparatus such as beads, numbers lines, empty number lines and so on, so that learners gain confidence in using counting as a means of calculating. Counting strategies lay the basis for learning addition and subtraction, initially using strategies like ‘counting all’, then ‘counting on’ or ‘back’ (Carpenter, Moser and Romberg, 1982; Fuson, 1992). With experience, and the acquisition of a growing repertoire of number facts, learners develop the competence to compute without a reliance on counting strategies.

In summary, in the early years of learning mathematics, students move through a number of stages in the shift from counting, and an understanding of numbers as reflecting numerosities, to calculating, and the conception of numbers as objects which can be manipulated according to certain laws. The question for the analysis that follows is how this shift is best described.

Specialising pedagogic text and time

The analysis of data presented in this paper is premised on the understanding that the shift from concrete to abstract reasoning depends primarily on the sustained specialisation of pedagogic text over time. Texts make up the semiotic system which teachers mobilise in classrooms when they teach, and refer to any utterance, object or inscription which teachers communicate or otherwise make available to learners in the practice of teaching. Pedagogic texts include verbal communication by teachers, inscriptions on the blackboard, bodily movements recruited for the purposes of teaching, and various forms of apparatus, such as counters, pictures and number charts. Specialising strategies, (following Dowling 1998), work to draw pedagogic texts away from the particular, the concrete and every day, towards more abstract forms of representation. Pedagogy in the numeracy classroom does this in three ways: by rendering the *content* more abstract; by specialising the *modes of expression* used to communicate this subject matter; and by rendering more abstract the *forms of representation of number*. An example of what we mean by specialisation of pedagogic text is provided in one of the Grade 1 classes we observed in which the teacher consolidated learners' understanding of the notion of 6. Learners were provided with counters to count out 6, and were then asked to partition 6 in various ways to show different quantities that could be added or subtracted to produce 6. She also encouraged children to write 6, and modelled for them how to do this. By providing multiple representations of the number 6, the teacher assisted learners to grasp the invariant, 6, and to represent it in symbolic form. She abstracted from various instances to consolidate the notion of '6' (specialisation of content); she used increasingly abstract forms of representation, from counters to numerals (specialisation of representation), and in the language she used in making these moves she buttressed learners' efforts to grasp the notion of 6 (specialisation of mode of expression).

Because of constraints of space we focus in this paper only on specialisation of content and mode of representation, and leave for future papers a discussion of the specialisation of language, or mode of expression.

In the analysis of data, then, we were interested in the extent to which teachers specialised pedagogic text from the counting of concrete objects, to calculating-by-counting (with, and then without the aid of apparatus), and finally to calculation using symbolic, syntactical mathematical language. We expected that utterances, written texts, apparatus of various kinds, and different modes of representation would become specialised as pedagogy moved learners away from concrete experiences to an understanding of abstract mathematics. In analysing the specialisation of text, in relation both to content and modes of representation, we therefore looked for shifts from counting, to calculation-by-counting, and to calculation.

Semantic density

We were interested in the specialisation of text, but also in the relationship between text and time. Teachers make decisions about how much time to spend on a topic, how they order topics within and across time periods, how much time they allocate between whole class teaching, group work and individual work, how much time they spend speaking themselves and how much time they allow learners to speak, how much time they allocated to speaking and writing, and whether the allocated time is filled by all learners working at the same pace, or at different paces. Specialising pedagogic time in the teaching of numeracy entails the purposeful deployment of time in order to deepen learners' awareness of numbers and the number system. A highly specialised task can be spread over a long period of time, whereas a low level task can be allocated a small amount of time. Or, looking at this in a different way, given the same period of time, we can stretch tasks of different levels of abstraction across it.

The specialisation of text and time thus both contribute to *semantic density* (Ensor, 2009) – the distribution of text across time. The notion of semantic density grasps simultaneously the twin concerns of specialisation of text, and its distribution over time. It is for this reason that we talk of the specialisation of text and time together in the discussion that follows. High semantic density is achieved via the distribution of specialised text across concentrated periods of time: levels of semantic density can be reduced by localising pedagogic text and/or expanding pedagogic time.

Coding the data

The nine teachers in our sample were observed for two lessons each, one in 2004 and another in 2006. We began the task of analysing the classroom data by dividing each transcript into a set of pedagogic tasks where a task was defined as a segment of a lesson which was constituted around a single goal or theme. A single task could entail a number of activities which were semantically intertwined. For us, punctuation of a classroom text into tasks was usually signalled by the teacher as she changed focus from one topic to another, or, within the same topic, changed the mode of classroom organisation. So for example the switch from acoustic counting to the exploration of the number line was taken to mark the end of one task and the beginning of another. Similarly, part of a lesson devoted to whole class teaching of multiplication, followed by individual work by learners on the same topic, was regarded as two tasks, the first based on whole class teaching, the second on individual work.

As we have pointed out above, learning number entails learning to count and gaining mastery of the principles enumerated by Gelman and Gallistel. This provides a platform for learners to learn to calculate using counting strategies, and then to move beyond this to learning to calculate using symbolic representation, with a diminishing role for counting. As we have indicated, the move from the concrete to the abstract in relation to number in the Foundation Phase entails three parallel sets of specialising strategies, in relation to content, to mode of representation and to mode of expression.

Specialisation of content

The specialisation of content in the teaching of number entails a shift from counting, to calculating-by-counting, to calculating without counting. The categories presented below emerged from the analysis of classroom data, as well as from engagement with literature on early number acquisition. They are not presented here as a full or ideal sequence in the learning of number.
By *counting* we refer to the presentation of a range of tasks such as:

- *Acoustic (or oral) counting*, which encourages learners to memorise number sequences and number patterns. Angilheri (2006) argues that counting forwards and backwards, counting in 1s, 2s, 3s, 5s etc. develops an understanding of patterns that assist in early addition and subtraction. Acoustic counting was present in all grades we observed, accounting for approximately 7 per cent of the pedagogic time of each grade. Examples include counting forwards in 1s, 2s, 3s, 5s, 10s, 25s and 50s, and backwards in 1s, 2s, 5s and 50s (seen in only one lesson).

- *Counting out object*s entails mapping a number sequence on to a set of objects, which includes counting animals on a poster, counting members of a family depicted in a picture, counting out a set of objects such as counters, stone, crayon boxes, beads, dots and tallies. We have included in this category a Grade 2 teacher's division of an apple and a loaf of bread in order to introduce the notion of a fraction as a result of sharing.

- *Producing and recognising a written number sequence.* Acoustic counting enables learners to memorise number sequences, which they need to be able to recognise and to reproduce in written form.

- *Locating numbers on a number line or chart and learning number facts*, which includes identifying numbers on a number chart, finding numbers closest to a given number, bigger or smaller than a given number, or between two given numbers. It entails the use of expanding number cards to represent numbers.

Calculating-by-counting tasks use counting for the purposes of calculation. For example, in one of the lessons we observed the teacher provided a drawing of three families of different sizes and asked learners to rank the families in size, quantifying the differences between them. Various counting strategies were used to achieve this.

Calculating without counting tasks entail adding, subtracting, multiplying and dividing and do not rely on counting but rather on memorised number facts (such as number bonds and times tables). This includes the addition of two-digit numbers using expanding cards. Treffers (cited in Menne, 2001) identifies three increasingly complex levels in mental solution strategies: calculation by counting, structured calculation, which entails calculation without counting but the use of suitable models (such as, for example, an empty number line), and formal calculating, which relies on mathematical language and conventions and does not require structured materials or models. Since we observed very little work in calculation without counting we have not refined the subcategories further, along the lines which Treffers suggests. The progression we have outlined above, from counting, to calculation-by-counting, to calculation without counting, entails the increasing specialisation of pedagogic content. We now consider the specialisation of forms of representation.

Specialisation of mode of representation

The move from concrete representations of number, to symbolic representations, again reflects increased specialisation of pedagogic text.

The following forms of representation of number[4] were used in classrooms:

- ***Concrete*** apparatus which entailed the manipulation of physical objects such as fingers, bodies, money (real or plastic), crayons, matches, boxes of groceries, plastic pigs, pegs. We also include here counters, cards or beads (single or string). This apparatus was used for counting and for calculation-by-counting strategies.

- ***Iconic*** *(images of everyday context – realistic depictions)* apparatus included photographs, cartoons, or drawings (for example, worms, washing lines). This apparatus was used in the same way as concrete apparatus but could not be manipulated in the same way.

- ***Indexical*** *(indexes everyday contexts – generic rather than realistic depiction of everyday contexts)* apparatus featured drawings of sticks, tallies, dots, circles and other shapes to represent everyday objects. This apparatus was used for counting and for calculating-by-counting tasks.

- ***Symbolic – number-based*** *(use of numerals to represent numbers)* apparatus including number lines (structured and semi-structured), number charts, number cards. This mode of representation supported calculation without counting but could also be used for calculation-by-counting tasks.

- ***Symbolic – syntactical*** *(use of mathematical notation to produce mathematical statements).* This mode of representation is abstract, and entails the deciphering and production of mathematical statements. It relies on known number facts and facts which can be derived without counting.

- ***No representation used*** – this refers to tasks which learners were asked to carry out which did not entail the use of modes of representation. This included acoustic counting, and mental arithmetic. Representation in this case was internalised.

These representations of number shift from the concrete, here-and-now of counting using fingers and other objects, to the use of tallies and other marks,

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4 This description of different forms of representation draws loosely on Dowling's (1998) analysis of modes of representation in a mathematics textbook scheme.

to the use of mathematical notation to undertake calculations with or without reference to empirical situations. We would expect that as learners progress from Grade 1 to Grade 3, concrete, iconic and indexical apparatus would give way to symbolic forms of representation, in the first instance in the representation of numerosity by numerals, and then the production of mathematical statements in symbolic form.

The COCA project

Nine Foundation Phase teachers teaching in three different semi-rural, poor schools in the Western Cape constituted the sample for the research reported in this paper. All teachers speak isiXhosa as their first language, the home language of the majority of learners in the classes of the teachers was isiXhosa, as was the medium of instruction. Six of the nine teachers had experience in teaching Grade 3; the other three had taught Grade 1 and 2 and in one case, Grade 5. All teachers were female, older than 30 years of age and qualified to teach at the Foundation Phase level. Two of the teachers had Bachelor degrees, and two of the teachers had the lowest level of teacher qualification – a matric plus a three year diploma. The teachers varied in terms of their teaching experience, between 5 years and 25 years. The teachers' classes were on average large, with as many as 57 learners in one class. Only two classes fell within the national teacher : pupil ratio norm of 1:40.
The teachers were video recorded teaching a lesson in 2004 and again in 2006. We requested that we be invited to record lessons on number but it turned out in the end that two of the lessons were on measurement. We have included these in our analysis as both entail some number work. Where tasks did not entail the use of number in some way, we have classified these as 'other' in the analysis that follows.

Analysis of data

Specialisation of content

In total, the lessons of all three grades, across both 2004 and 2006, totalled 879 minutes. We observed seven Grade 1 lessons, five Grade 2 lessons, and six Grade 3 lessons. The transcripts were divided into tasks, and these then further divided into three primary modes of classroom organisation – whole class activity, group work, and individual work. Whole class activity involved the teacher engaging the attention of all learners, focussed on a common task or set of tasks. Group work entailed the distribution of a task, or tasks, to a group. We will discuss further below the particular issues that arose in the distribution of tasks within groups. Individual work entailed engagement by learners on tasks, working alone.

Table 1: Distribution of time to classroom organisation

	Grade 1	Grade 2	Grade 3	Total
Whole class activity	223 mins 64%	162 mins 69%	193 mins 65%	**578 mins 66%**
Group work	40 mins 12%	28 mins 12%	58 mins 20%	**126 mins 14%**
Individual work	85 mins 24%	45 mins 19%	45 mins 15%	**175 mins 20%**
Total	**348 100%**	**235 mins 100%**	**296 mins 100%**	**879 mins 100%**

Table 1 shows that across the grades we observed, whole class teaching absorbed approximately two thirds of class time, group work approximately 14 per cent and individual work around 20 per cent. This suggests that the ways in which pedagogic time was utilised during whole class teaching was crucial to learners' success as they were dependent on teachers for effective communication of mathematical ideas.

We then analysed each mode of organisation in turn, looking at counting tasks, calculation-by-counting tasks, and calculation tasks. In practice the two ends of this spectrum – that of counting on the one end and of calculation on the other – were much easier to detect in classroom activity than the shift from counting, to calculation-by-counting, and the shift from calculation-by-counting to calculation without counting. We differentiated tasks on the basis of how teachers set them up and the strategies they encouraged learners to use. So in two Grade 3 lessons, the teachers gave learners word problems involving two digit numbers and provided counters to assist them in doing this. We classified this task as calculating-by-counting. In another Grade 3 lesson, the teacher gave learners two-digit word problems, but indicated that she expected these to be solved through partitioning and adding. We have no record of whether learners in fact used fingers or tallies to support their efforts in this lesson, but we classified this as a calculating task. As we will show, this was uncommon in the lessons we observed.

The table below shows the allocation of pedagogic time across the three categories of task, aggregated across all three grades, and for all modes of classroom organisation.

Table 2: Total allocation of time to categories of task

	Minutes	**% of total pedagogic time**
Counting	306	35%
Calculating-by-counting	471	54%
Calculating	26	3%
Other	76	8%
	879	**100%**

This table shows that 89 per cent of total pedagogic time was spent on counting or counting-by-calculating. Grade-specific data shows that in Grade 1 there was a heavy emphasis on counting, with relatively little time on calculating-by-counting. By Grade 2 there was greater evidence of the latter. By Grade 3 we see a relative increase in the proportion of time spent on calculation-by-counting and calculation, and a decline in the amount of time spent on counting. This is as we would expect the situation to be. However, there was relatively little pressure, in Grade 2 and 3 in particular, towards calculating without reliance on counting. In two Grade 3 classes, for example, learners were asked to solve word problems involving addition and subtraction and were given counters to assist them to do this.

This suggests that while some degree of specialisation of number content occurred across the three grades, the amount of time spent on calculating was very low, and occurred only in Grade 3. Very little attempt was made by teachers to encourage calculation without counting in the lower grades.

Specialisation of modes of representation

In order to obtain a measure of the specialisation of modes of representation across grades we counted the presence of apparatus used in each lesson, per task. This means that if fingers, beads, counters and stones, for example, were all used for a single counting task, we noted a single occurrence of the concrete. Table 3 therefore registers the presence of a particular mode of representation, but not the extent of its use. We have counted apparatus for an entire lesson, and have not differentiated according to form of classroom organisation. We have also excluded from consideration tasks which have been classified as 'other'. We obtained the following table:

Table 3: Forms of representation and number of tasks for which used

	Counting			Calculating-by-counting			Calculating			TOTAL
	Gr1	Gr2	Gr3	Gr1	Gr2	Gr3	Gr1	Gr2	Gr3	
Concrete	8	4	2	7	10	10	-	-	-	**41**
Iconic	8	-	1	5	3	-	-	-	-	**17**
Indexical	3	-	-	2	-	1	-	-	-	**6**
Symbolic	17	5	7	6	9	3	-	-	3	**50**
Syntactical	-	-	-	9	2	9	-	-	3	**23**
None	1	5	5	1	1	6	-	-	1	**20**
TOTAL	**37**	**14**	**15**	**30**	**25**	**29**	**-**	**-**	7	**157**

This table suggests that concrete apparatus for counting, and for calculating-by-counting, is visible in all three grades, with sustained use through Grades 1, 2 and 3. The relatively low employment of indexical forms of representation surprised us, given the ubiquitous use of tallies for the purposes of calculation that have been reported on by Hoadley (2007) and Schollar (2008), and which was manifest in our own assessment data. Evidence from our studies suggests that across the grades, teachers favoured concrete apparatus over the use of indexical marks to stand for them. While the representation of number as numerals (the symbolic) was common across the grades, the use of written mathematical statements was less frequent, and more visible in Grade 3. So some specialisation in modes of representation occurred across Grades 1 to 3, but not to the degree we would have expected. As the Table shows, and which we will discuss further below, teachers prioritise counting and calculating-by-counting and the use of apparatus to support these activities well into Grade 3, which has a negative impact on the conceptual level of the number work offered to students and the use of time.

Specialisation of text in time

Having considered the specialisation of text, in terms of content and mode of representation, we were then interested in the distribution of text across time.

We chose as a measure for this the density of computations across grades. We went through all of the tasks and aggregated all the computations, whether these entailed counting or not, and irrespective of whether learners were asked to complete these computations in whole group teaching, group or individual work. We took a very broad and generous view of this, counting every computation regardless of whether it was oral or written, whether it entailed simple one-digit addition or a word problem involving two digits. Where different problems were given to different learners, with the expectation that each learner should complete only one, we counted all of the computations set for the class. We found the following:

Table 4: Distribution of computations over time

	No. of computations	Computations/time x 60
Grade 1	20	4 comps per hour
Grade 2	19	5 comps per hour
Grade 3	55	11 comps per hour

This means that at Grade 3 level, learners were exposed to approximately 11 computations per hour, and slightly less than this in lessons lasting the average length of around 50 minutes.

The quantitative data we have presented above suggests that some degree of specialisation of text, both in terms of content and mode of representation, occurred over Grades 1 to 3. However, Table 4 raises concerns about the rate, and the extent, to which this took place. We therefore extended this quantitative analysis with a qualitative dimension, to highlight examples from classroom practice which underscore the concerns we raise.

As we have indicated above, in all three grades, the majority of pedagogic time was spent on whole class teaching, and most was spent on counting or calculation-by-counting. Most of the apparatus we have recorded on Table 3 above was thus used in a whole-class context, as a tool for demonstration by the teacher rather than for manipulation or handling by learners, and as a support for counting. This impacted not only on the degree of specialisation of the text, but also on the way in which time was used. In general the use of apparatus anchors experience in the local and particular and explicit specialising strategies are needed to facilitate the move to abstraction. In our research there was limited evidence of these strategies, a problem which was compounded by the fact that the use of apparatus also consumed a significant amount of class time in the classrooms we observed. Setting up tasks which involved apparatus took time, both to assemble and to explain, an expenditure of time which was commonly in inverse relation to the mathematical demands of the task. In a Grade 3 class, for example, the teacher took the class outside and set up a task much like skittles, in which one learner per group took turns in throwing a ball at plastic bottles lined up across one side of a court yard. Learners then counted out all the bottles, then the bottles that fell, and were then expected to generate a sum which represented this.[5] This activity took 25 minutes, half of an entire lesson, and in the end generated three subtraction sums (one per group) involving two digits. In another Grade 3 class, which was devoted to measurement and the concept of volume (although the term 'volume' was never used), the entire lesson was spent by the teacher pouring water from one container into various others. Her intention was to illustrate the standard unit of measurement of volume, the litre, and subunits of this. However, she did not use accurate measuring equipment and the outcomes were incorrect on a number of occasions.

The use of apparatus undermined the specialisation of text and the efficient use of time in whole class teaching contexts, as well as in group contexts. Across all three grades we encountered eight instances of what we have classified as group work. This entailed the setting of a task which the group was supposed to solve together. In every case of group work we observed, only one set of apparatus or writing material was supplied for the entire group. This meant that one learner in the group completed the task while other learners looked on. Setting up group work in this way invariably entailed the use of some kind of apparatus, and took a very long time to get underway. In every instance of group work the teacher went from group to group, explaining and re-explaining what needed to be done. Yet all of the tasks set as group work projects were of a low mathematical level. In one Grade 2 group, for example, learners were asked to paste matchsticks in groups of 3 on a poster. One learner pasted the matchsticks while the others observed. This had followed a demonstration by the teacher on the board of grouping in threes, and the learners were required to simply reproduce, and not in any way extend, what they had been taught. Another group in the same lesson was expected to place cards on every third number on a number board; one learner

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5 This has been classified as a whole class activity here, rather than group work. Although the groups went in turns to throw the ball, the remaining learners watched and waited their turn, with nothing else to do. One sum for each group of learners was the only inscription made during this lesson.

was required to undertake this while the others observed. A third group was required to group pegs in arrangements of 3 on a peg board – again, one learner carried out the task while the others looked on. The mathematical requirements of these three tasks were trivial, and yet none of the groups managed to complete what they were asked to do by the end of the lesson. The teacher went from group to group explaining not the mathematics involved, but what the learners needed to do with the apparatus.

In another, Grade 1 class, learners were expected to paste numbers from 1 to 10 on a poster. One learner accomplished this while the others looked on. In another Grade 2 class a large group of around 10 children were tasked with cutting one orange into 4s, with a very blunt knife. In all these cases a great deal of time was devoted to the activity and boredom inevitably set in, giving rise to discipline problems as learners bickered over the sharing of apparatus.

Of the nine group work tasks set altogether, four were completed in the lesson and involved some kind of plenary feedback. It was not uncommon to find a lesson ending without any work by learners being undertaken at all, whether in a group or individual context.

Table 1 shows that very little class time was spent on individual work. There were ten tasks involving individual work over all the classes we analysed, which varied in mathematical level. In a Grade 2 class we observed learners pasting paper pigs into two circles drawn in their exercise books to represent sties, a task which reproduced almost exactly content that had already been taught on the board. The lowest level of task, in terms of specialisation of mathematical text, involved the colouring in of different size containers (Grade 1). The task of greatest complexity offered, to Grade 3 learners, was adding two-digit numbers by partitioning and adding. While most of the individual tasks were intended to provide opportunities for learners to write, this often did not happen as the lesson came to an end before they could complete their work. Of the ten individual work tasks set, none were completed in the lesson so as to allow for some kind of plenary feedback.
The use of apparatus expended a considerable amount of pedagogic time as teachers set up whatever it was that they wished learners to do. But even when apparatus was not used, the setting up of tasks took time. This was compounded by the number of activities which involved group work, which took time to set up. All teachers spent some time reading out the problems set, highlighting component parts and breaking tasks up into subtasks before learners were able to proceed.

The data presented above on the specialisation of text in time shows a very low rate of transmission occurring in the classrooms. The extensive use of apparatus in whole class and group contexts entailed protracted periods devoted to setting up tasks, at the expense of learners engaging in worthwhile mathematical activity. Learners were exposed to a low number of computations per hour. By Grade 3 there remained a heavy reliance on counting as a calculation strategy. The semantic density of the lessons, or the specialisation of text across time, can thus be characterised as low. In addition, the fact that students accomplished very few tasks individually means that the experience of the pedagogy (and its density) was uneven across different learners in the same classroom.

Learner performance

The students in all of the teachers' classes were tested at the end of each of three successive years over the course of the COCA project. The tests were benchmarked against the national curriculum, and covered three identified knowledge areas of number: visual, symbolic, words, and a combination of these. The tests were also designed to address different skills categories: resultative counting, representing numbers and calculations. The data shows very low performance levels of students in all classrooms. There was one Grade 1 teacher (who, interestingly, was the least qualified in terms of formal qualifications and had been teaching for only five years) whose students scored an average of 50 per cent over all three years of assessment, with a mean score in year 3 of 65.3 per cent. In addition, the student of one Grade 3 teacher scored on average 51.7 per cent across all three years. No Grade 2 teacher in any of the three schools we worked in achieved a learner mean score of over 50 per cent.

In administering the test, a qualitative response sheet was used to identify learners and capture learner strategies in solving problems. This observational research found that overwhelmingly, learners showed no strategies for solving problems, but simply wrote down a response. Where strategies were used, we found the use of tally counting predominated across Grades 1, 2 and 3. Grade 3 learners still used tally methods with no evidence of structure-based strategies such as group counting or other more formal approaches.

Conclusion

Our research examined the specialisation of pedagogic text in time in the Foundation grades in three schools which cater for students from very poor backgrounds. We were interested in the ways in which text was specialised, through the move from counting through to calculating without counting; through shifts in the use of apparatus, through different modes of representing numbers and through the steady specialisation of language use in classrooms. We also considered the manner in which text was specialised within time. We found that while shifts from concrete to abstract modes of reasoning were evident in all classrooms we observed, this did not happen at the pace or at the depth that learners require in order to move on to more complicated arithmetic operations in the intermediate years. Teachers simply did not present learners with enough mathematics, at a sufficiently complex level.

The concept of semantic density highlights the low conceptual level of the pedagogic text, as well as the low rate of transmission. In combination, the relatively low level of specialisation of text over time has severe implications for whether and how learners acquire an understanding of number, and this is borne out in the assessment data presented.

In their efforts to specialise text in time we believe that teachers are hampered by the National Curriculum Statement and its associated recommendations. We turned to the Statement for guidance on what content should be covered at what level across the three grades, and were not able to find adequate guidance. On the face of it, the teachers we observed seem to comply with the stated requirements of the NCS, but these do not specify in sufficient detail what should be covered, at what level, across what period of time. The proposal of the Foundations for Learning initiative to provide these milestones will make a significant contribution to solving this problem. We are concerned, however, that the FFLC continues to place emphasis upon group work, and the use of apparatus, without emphasising that these are strategies that teachers may use to achieve pedagogic ends, and do not constitute ends in themselves. In the lessons we observed, the use of apparatus, and of group work, had the effect of localising pedagogic tasks, and dissipating pedagogic text in time, to the detriment of learners' progress. By inscribing the use of group work in the way in which the FFLC does, we run the risk of further embedding the practices which we have described, which hinder the mastery of number.

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