Complementarity, mathematics and context

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Abstract

In this paper the concept of complementarity is developed as a theoretical tool for analysing a mathematics pedagogy, particularly one from a critical perspective that foregrounds the issue of context.[1] Complementarity has its origins in the work of the physicist and philosopher Niels Bohr. Although it arose out of a dilemma in quantum physics, it is now widely used in diverse fields and disciplines including mathematics and mathematics education, often as a justification for bringing together irreconcilable conflicting but necessary positions or theories. I explore a particular interpretation of complementarity derived from the work of Michael Otte to explain the empirical evidence produced in mathematics classrooms. The relationship of 'opposite and complementary' that is captured in this conception is recruited for understanding the link and disjuncture between mathematics and context. Given the strong imperative of the new South African curricula reforms to teach a more contextualised mathematics, the challenges this poses for teachers and learners needs to be theorised from many different perspectives, especially if the espoused social, cultural and political goals are to be achieved, and the possibilities for equity and social justice are to be realised through such curricula.

Introduction

It was a particularly low moment in my doctoral studies when I first came to consider complementarity. I was exploring the question of 'what happens in a mathematics classroom when student teachers attempt to realise a social, cultural, political approach to a mathematics curriculum, particularly one that integrates a critical perspective', and had begun to identify pairs of themes but was unable to theorise them further: democracy and authority; freedom and structure; mathematics and context; equity and differentiation; potentiality and

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1 An earlier version of this paper was presented as a plenary at the 14th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), University of Pretoria, 9–12 January 2006.

actuality (Vithal, 2003).[2] I was especially drawn to the idea of complementarity through a seminal article by Brousseau and Otte (1991) because it allowed me to begin to explore these dual concept themes as 'opposite and complementary', which seemed to capture the complexity of mathematics teaching and learning that I was seeing in the data and did so in a way that did not reduce explanations to deficit analyses of teachers' and learners' work.

In their aptly titled chapter, 'The fragility of knowledge' in a volume on *Mathematical knowledge: its growth through teaching* (Bishop, Mellin-Olsen and Van Dormolen, 1991), Brousseau and Otte

> attempt(s) to demonstrate a different aspect of the fact that the human being, on the contrary, is at the same time both the subject and the task of cognition or the source and the object of activity.
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> This two fold necessity leads to the apparition of a whole series of pairs of concepts: insight and action, intuition and formalism, and so forth, the character of which we have tried to show as both paradoxical and necessary, opposite and complementary. These oppositions are the source of the fragility of the act of knowing and the difficulties in the transmission of knowledge. (p.35)

The pairs of dual concepts I had identified could be seen to be forming an opposition and an alliance, working antagonistically and yet also in co-operation with each other. They could be understood as being separate from each other but also contained in each other. The idea of complementarity, offered a way to seize the essence of the meaning of the concepts and of their relationship – a way of analysing and theorising about what was happening in the classroom that grasped a more sophisticated and deeper understanding of the dual concepts in the themes, and of the concepts themselves. In this paper I focus on one pair of concepts – mathematics and context – a concern that has gained increasing importance with the rise of socio-political, historical and cultural dimensions of mathematics education.

Brousseau and Otte (1991) might have had in mind quite a different classroom. However, the importance of this idea of complementarity took on even greater significance because the complexity of the classroom I was

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2 For this paper I draw substantially from this earlier work to, firstly, offer a consolidated account of the development of the notion of complementarity (Vithal, 2003, Chapters 8 and 9, pp.301–359) and then to illustrate and consider its continued possibilities in understanding and acting on the relation between mathematics and context especially in light of the new South African mathematics curriculum reforms.

researching increased several times when the goals of mathematics teaching and learning were broadened, as they had been, to bring a social, cultural, political approach that integrates a critical perspective and a concern for issues such as equity, gender, race, and social justice into the classroom. Brousseau and Otte (1991) illustrated implicitly the notion of complementarity – the complexity in the contradictory and complementary nature of their elaboration of mathematics teaching and learning through the 'paradox of the didactical contract'.

It may be useful to examine this paradox to understand the foundational dilemmas and tensions that teaching and learning embody because inherent in this contract is both the failure and success of teaching and learning mathematics. According to Brousseau and Otte (1991), the didactical contract between teachers and learners is that "the teacher is obliged to teach, and the learner is obliged to learn". This contract cannot be negotiated, it cannot be controlled by either teacher or learner and nor can it be ignored. It "must be honoured at all costs for otherwise there will be no education. Yet to be honoured, the contract must be broken because knowledge cannot be transmitted ready made". (p.18)

So the 'paradox of the didactical contract' between the teacher and learner arises because the learner is deprived of the conditions for learning and understanding when both the problem and the means for its solution are communicated by the teacher. One way to resolve this difficulty may be to open the situation so that learners can choose and decide for themselves. But this leads to a second paradox:

> To get involved in an open situation and to gradually gain control over it apparently represents, on the one hand exactly the conditions necessary for the acquisition of new insight and knowledge. On the other hand one may consider these activities to be the usual employment of already acquired knowledge. As the person solves a problem inherent in a situation, he obviously has all the knowledge that is necessary for that purpose. The fact that he learned something from the situation is manifest by his failure to solve the problem. The knowledge is the prerequisite as well as the result of the problem solving activity (p.34).

Are we trapped in the didactical contract? Perhaps not, because what is demonstrated here is exactly that the failure of the contract represents also its success. The didactical contract set up (implicitly or explicitly) between teachers and learners embodies the complementarity of the paired themes – democracy and authority, differentiation and equity, freedom and structure; actuality and potentiality – whatever meanings these notions come to have in a classroom. In particular, the complementarity of mathematics and context, which is the focus of this paper, recognizes that the didactical contract in a *mathematics* classroom between teachers and learners specifies that 'the teacher is obliged to teach *mathematics*, and the learner is obliged to learn *mathematics*' and insists that this is honoured at all costs – but is it or should it be? And what of teaching and learning about context?

Some early data that lead to complementarity…

Before offering a detailed discussion of exactly how complementarity was interpreted and developed, it may be useful to give a glimpse of the initial data that led to the idea of complementarity, and provided the key to unlock the analysis in a way that allowed an examination of the dynamics and movements in a mathematics classroom especially from a critical perspective. The student teacher, Sumaiya, whose work I theorised as she attempted to realise a social, cultural, political approach to the mathematics curriculum, involved her diverse grade 6 mathematics class in a number of projects which they undertook in groups.

In Episode A below Sumaiya poses a question to a group of girls who had just presented their project – a mathematical newsletter – to the class. Even though the group had developed and engaged a number of activities, many of which they had interpreted mathematically, they claimed that they could not find any mathematics. What happens to the mathematics, why are they unable to see it in the world around them?

Episode A

#172. The difficulty of looking for and locating the mathematics:

Nikita: It was very difficult to relate everything to maths but we tried our best.

Sumaiya: So how did you all make that link because initially, I know you had a lot of English stuff and not much maths till I brought that to your attention?

Vasentha: We had to relate to maths by putting numbers and…

Sumaiya: But why originally, did you make that misconception, relating it to English only?

Neeta: Ma'am, because basically, **everything that is around us, almost everything is not related to maths. Almost everything around the world hardly has any numbers**.

Vasentha: **Its very difficult to relate everything to maths**.

(Newsletter Group 2 presentation on Day 9)

In Episode B a group working on a project investigating the money spent on their education used this as an opportunity to question the school's use of their school fees for constructing a shelter (a hall-sized roof without walls).

Episode B

#233. Presentation of project ideas, learning about backgrounds:

Devan: We had already written out our school budget. Also getting to know our parents' salary. We are doing monthly work out and trying to find out whether our school fees should be higher or brought down. Some people have very little money to pay for food.

#234. Questioning the use of school funds and the 'structure' (a hall-sized roof):

Mohan: I don't think the structure is very important. So much of money is spent on this when our toilet facilities need to be improved. Need money for computers. We shouldn't worry how our school looks, rather on our education.

Teacher: Can I disagree with you immediately? We have very, very hot sun and so much of the time we cannot have all the activities. You know the play you watched 'Trouble with Andre', you paid R1.50 for the 600 people. It will cost more in terms of money, theatre. The disadvantaged students will then not benefit. You need to discuss how often this structure is used… **Do not get side tracked. We are doing it in a graph form**.

#235. Mathematics saves the teacher:

Sumaiya: What graph are you using?

Devan: We are still deciding. We want to use the pizza graph and then make a summary.

Sumaiya: Are you going to draw one graph for all the pupils or are you going to use different graphs for each individual pupil?

Devan: We are going to take everybody's points and draw one big graph and explain to the class. Is that ok?

(Cost of Education Group 3 from the lesson on Day 4)
(Vithal 2003, p.270–271, emphasis and brackets added)

Both the student teacher and the class teacher force a move out of the context and into the mathematics – a graph drawing activity – to change the course of the discussion. Complementarity offered a particular means for understanding the relation between mathematics and context: when one seemed to appear or be engaged, the other disappeared; the difficulty of keeping both present and visible; and the ways in which context and math sometimes worked to support each other but also served to challenge or block each other.

Complementarity

The notion of complementarity has its origins in the work of the physicist and philosopher Niels Bohr. This idea emerged from an empirical reality that is metaphorically similar to the way in which I explored the idea. I too am concerned with explaining empirical data – classrooms and people rather than atoms and particles. Complementarity, offered a way out of a dilemma that arose in quantum physics. The problem was, as can be found outlined in any basic physics textbook, that no single concrete mental image, combining the features of both wave and particle at once, is possible in the quantum world. A solution through the principle of complementarity, may be explained as follows:

> The wave and particle aspect of a quantum entity are both necessary for a complete description. However, the two aspects cannot be revealed simultaneously in a single experiment. The aspect that is revealed is determined by the nature of the experiment being done.
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> (Halliday; Resnick and Krane, 1992, p.1063)

Complementarity offers a powerful means for dealing with the problem of understanding the development and co-existence of significantly different, even opposing theories, explaining the same phenomenon. For example, the phenomenon of light may be understood through two separate explanations. In some experiments light behaves like a wave and in others it behaves like a particle. There is no single experiment that enables an interpretation of light as a wave and as a particle at the same time. Is light a wave or a particle? It seems that it cannot be both; and it is neither. When experienced in one way the other is excluded. The phenomenon cannot be understood in its full complexity through the one, and equally, both interpretations cannot hold at the same time. The theories appear to be in opposition to each other and yet complementary to each other. Most importantly, both are needed to understand the phenomenon fully.

Complementarity is applied in a wide diversity of fields from art, literature to economics. An internet search yields several thousand entries. The idea of complementarity has also been invoked in mathematics and mathematics education by several authors (see for example, Kuyk, 1977; Steiner, 1985; Mellin-Olsen, 1993; Bartolini Bussi, 1994; Ernest, 1994; Sfard, 1998). Often, it is not elaborated, but used as a justification for bringing together irreconcilable conflicting but necessary positions or theories in mathematics and mathematics education. However, its most substantial development has been in the work of Otte (1990, 1994).[3] According to Otte (1994), this fundamental principle which appears in the concept of complementarity, is foundational to every philosophy of mathematics. For Otte, complementarity represents a basic perspective in our coming to understand and provides a way of speaking about our means for understanding as being insufficient. If we take any one perspective, then we exclude another. This does not mean that the other is not present but that when we experience the one the other is excluded.

Otte offers different examples for demonstrating relationships in complementarity both in mathematics, in for example intuition and axiomatic thinking (Otte, 1990), but also more generally (Otte, 1994) such as: intentionality (consciousness) and communication; function and structure; passivity and activity; and so on.

> We could say that it is exactly the heading 'society as a laboratory' that embodies a universalisation of the complementarity of form and historicity; of structure and process, and so forth that gives another character to the whole problem (of the strict separation of subject and object which is challenged but also required), because we are simultaneously subjects or creators, as well as the ones who are affected by the creations.
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> Otte, 1990, p.60, (brackets added)

Otte casts his net very wide in this application of complementarity yet he also elaborates it in terms of specific pairs of ideas. Complementarity is both a simple idea and yet also complex and difficult to get to grips with. A key example of complementarity explored by Otte (1994) is through the notions of *object* (or content), and *tool* (or concept). Object here does not have to do with aims but rather as something observed. Tool is an epistemic conceptual tool. Object and tool are separated but in the process of understanding they play a symmetrical role. Thus an equal status is given to both tool and object. Otte shows how not only are tool and object in a complementarity but also each contains a complementarity. Since both tool and object are active in knowledge production, we have two avenues for producing knowledge, through tool or concepts, and through objects or content. Objects and tools

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3 In its elaboration here, I rely on the translation and interpretation made in discussions between Ole Skovsmose and myself of Otte's work since much of it is not available in English. In both describing and interpreting the concept of complementarity from a chapter in Otte's (1994) book, I draw on these notes, which I have shared with Otte in personal communications. I am wary of the risk of a superficial or even 'misinterpretation' in this, yet it also contains the possibility for an alternative interpretation. I keep open for critique and further dialogue through this writing. For these reasons I give a more detailed account of my interpretation of Otte's writing.

never come to fit each other, they do not match. “Complementarity. . . means difference and relation simultaneously” (Otte, 1990, p.60). Since objects and tools are active, through the activity of both, new content and concepts are produced which are themselves active.

Complementarity between knowledge tools and knowledge objects is a real complementarity and not only a duality because neither could be described or defined without the other. Using this example, Otte specifies complementarity as constituting two main ideas.

- First, objects and tools are woven together. They presuppose each other. The one cannot be defined or described without the other.
- Second, objects and tools are contradictory to each other. They oppose each other. One does not directly show itself in the other.

The principle of complementarity expresses a fundamental condition for knowledge production. The two sources for knowledge production are knowledge tools and knowledge objects and they live and work in a complementarity to each other.

In my interpretation and use of the notion of complementarity, I do not intend to follow in Otte's footsteps as it were, but rather to use the underlying principles as inspiration for analysing the happenings of a mathematics classroom, particularly one in which we seek to realise a social, cultural political approach. The principle of complementarity may be featured in the interpretation and production of knowledge about mathematics classrooms and it speaks to how we are implicated in that process. I am proposing that complementarity offers a *theoretical analytical means for exploring a mathematics pedagogy*, especially from a critical perspective, because of the multiple goals and realities of a mathematics classroom that it seeks to engage. Through complementarity, the dual concepts in the themes such as mathematics and context could be seen to need each other, to develop each other, and in which one is required as necessary precondition for the other; yet also to exclude each other, to deny the existence of each other.

In the choice of the dual concepts themselves, it may be noted that they oppose each other, but they are not in direct opposition. We could, for example, take the opposite of democracy to be autocracy or authoritarianism. But this will once again return to an almost unitary conception because it reduces the complementarity of the concepts to a simple negation, which does not serve to explain the complexity of the theme. Complementarity subsumes a duality as its use in the themes points to a special relationship of *contradictions and co-operation* found in practice, in the classroom, in the attempt to realise the theoretical ideas of a curriculum approach, especially those that integrate a critical perspective within the framework of a largely traditional setting for teaching and learning mathematics. Furthermore, each of the concepts may be considered to contain elements of the other. So for instance, mathematics includes elements of context, it is produced and reproduced through and within different contexts but also does so on its own terms, operating in antagonism, and even incompatibility with context. Not all of mathematics can be taught and learnt through context. Contexts contain mathematical ideas and concepts but are also be valued on their own terms without reference to the mathematics. In this sense, the complementarity between the concepts, are also contained within themselves.

It is in exploring these contradictions that coexist in practice that we are better enabled to capture the complexity of the teaching-learning situation in theory. Rather than to set up unitary concepts that seem naturally to lead to uni-dimensional views and judgements about what teachers failed to do or what learners failed to learned, complementarity offers a different way of talking about and examining what happens in classrooms as teachers still continue to teach and learners continue to learn whatever it is that they teach and learn. That I choose this theoretical conceptual means, orientation, and ideological positioning is the result of a particular critical perspective that I brought to bear on all elements of the study – the question being posed, the methods/methodology deployed, the interpretation of data and approach to analysis, and its representation. As Otte (1990, p.58) points out, the ‘complementarist solution’

> shows that a certain solution to a problem will never force itself upon us, but that we have to choose the solution according to our view of the specific type of problem. Things never speak to us in an unequivocal way.

The analysis and theoretical developments made here are in complementarity with other perspectives that may equally be brought to bear on the same data and description. Indeed the success or failure, the growth or demise of particular tools or concepts and objects or content, and their relation, from a critical perspective, relies on, invites, and must provide a means for the possibility of alternative analyses and theorising. Complementarity as a core idea within a critical approach to research, theory, and practice, forces a humility and recognition that our knowledge is always incomplete, partial, tentative and fragile.

Mathematics and context

To explore complementarity in the theme 'mathematics and contexts' we might draw on existing analysis that have already been elaborated with respect to the content of mathematics as well as integrate other theoretical developments that focus on mathematics teaching and learning from a critical perspective. Complementarity is well established in the analysis of different forms of mathematical knowledge (see Kuyk, 1977). The complementarity between algebra and geometry as two forms of knowledge, each living their own lives besides each other and each having their own natures, yet not reducible to the other has been explored (Otte cited in Mellin-Olsen, 1993). Although each has its own theories, problems and ways of thinking, each has a presence in the other and represents a powerful method to illustrate and illuminate the other. Further Otte (1990) refers also, for example, to the relationship between arithmetic and geometry as a complementarity. He distinguishes between symbolic manipulation and conceptual argument. Within any graph drawing activity it is not difficult to see how the technical construction of graphs such as what scale should be used, and are the graphs correctly plotted on the axis, can get separated from a conceptual understanding of what is a graph and how does it feature in mathematics as a system. Learners need both to have the full meaning of graphs. But what of graphs and their relation to different realities – social, political, economic, historical or cultural; and their use for differing purposes – to explain, describe, predict, or justify decisions?

The idea that mathematics and context are in a complementarity relation is not new, particularly if we take context to include applications of mathematics. As Mellin-Olsen (1993, p.243) points out, "Knowledge of possible applications of some mathematical knowledge and the application itself is not the same knowledge as the mathematical knowledge itself". He argues that students are confronted with mathematical knowledge in various forms such as algorithms, models, and proofs; or as algebra and geometry, but "this variety is such that one form rarely can be reduced to another. Applied mathematics can not be reduced to theoretical mathematics." Nevertheless students have to be able to relate to each of these forms of knowledge and relate them to each other.

The question is how are or should these complementarities be handled in the classroom? Implicit in each of these is a particular conception of mathematics and of context, which needs to be broadened. For instance what are the implications for complementarity if we no longer consider mathematics but *mathemacy as a broad critical mathematical literacy* (Skovsmose, 1994) (not to be completely identified with the new South African Mathematics Literacy curriculum)? In a critical mathematics education, mathemacy brings together both a democratic competence and a critical competence. Mathemacy, elaborated as an integrated competence by Skovsmose (1994, p.117) "implies that the guiding principles for mathematics education are not any longer to be found in mathematics but in the social context of mathematics". In mathemacy, not only is mathematics found in contexts, but context also comes to reside within mathematics. Mathemacy comprises a mathematical, technological and reflective knowing but it is the component of reflective knowing that gives it its democratic and critical potential because it forces teachers and learners to engage social, ethical and political dimensions of the role and function of mathematics and its applications in society.

Six entry points to reflective knowing can be identified (Skovsmose, 1994; Keitel, Kotzmann and Skovsmose, 1993). In terms of this dual concept theme of mathematics and context we could posit a distinction between the first three, concerned with reflections that remain largely though not exclusively related to mathematics, which are: 1) selecting the mathematics; 2) executing the mathematics correctly and 3) trusting the reliability of the solution for the purpose; and the second three, in which the reflections relate more closely to the context – 1) the appropriateness of using mathematics in a specific context, 2) the broader consequences of the use of mathematics in a specific context and 3) reflecting on the reflection of the use of mathematics in a particular context. These two sets of reflections produce quite different competences and opportunities to participate in the *formatting power of mathematics* in context. That is, if as Skovsmose (1994, p.42) proposes, "mathematics produces new inventions in reality, not only in the sense that new insights may change interpretation, but also in the sense that mathematics colonises part of reality and reorders it", then the first set of reflections could be seen as being more aligned to inducting learners into a formatting process – to becoming 'critical formatters' while the second set of questions may be seen as producing the knowledge, skills and attitude to react to that formatting – becoming 'critical readers' of any mathematical formatting of context.

This means that reflective knowing includes two necessary but opposite forms of knowing – one inside mathematics, and the other outside mathematics – reflecting from some context back onto mathematics. When learners are inside the one, they seem unable to seriously engage with the other. The process of formatting located inside mathematics is in a complementarity with critically reacting to that formatting located outside mathematics. When concerned with the technical details of whether the graph is drawn correctly, or whether another graph would be a better representation, learners do not engage with questions like: is drawing the graph the best way of making my case or is the graph an authentic representation of my problem? In the project work presentation the teachers often gave priority in the mathematics class to the first set of reflections (as can be seen in Episode B). Thus a main concern in any critical approach must be with how to create better movement across these different forms of reflections. They are separate but they are also related. It is a difficult path to traverse back and forth as students participate, develop awareness and resist these shifts.

Skovsmose (1994) suggests that connections between these can be brought about through *challenging questions*. Challenging questions could create bridges between mathematics and contexts and across different mathematics or parts of a single context. The question, 'Have you learnt any mathematics as a result of doing your project?' asked by Sumaiya is a different challenging question from 'Has the mathematics helped you to deal with your project problem?', which was not asked. These questions would force learners into opposite directions of mathematics and context respectively. The first question leads to reflections on mathematics even though the project is mentioned. Learners' response, 'we have learnt to draw graphs', is a commentary on drawing graphs rather than on the project problem, keeping the focus on mathematical rather than the contextual components of the project problem, and remains firmly confined to the first part of the reflective knowing as distinguished above. The second challenging question, located in the second group of reflections, links to the project context, rather than to mathematics.

The project context could include a range of contexts. Learners drew graphs for the group or for each person in the group. Hence, the contexts in the projects remained largely at a local level of the individual and the group, rather than the class or school level. Nor did learners venture into the contexts at community, societal or global level. In order for this to occur it seems that challenging questions must be raised within contexts to link mathematics across the range of settings within contexts. This could also mean that different mathematics may be encountered from that already explored in the project, hence challenging questions within mathematics are also needed. This would require learners to look at different mathematics from that encountered in the project, and participate as critical formatters and readers of that mathematics in another context.

The teacher's role is crucial in managing the complementarity of mathematics and context. The teacher, through challenging questions, moves learners about within mathematics, but also directs them in and out of mathematics, and across the different contexts. However, learners also may and do pose questions, and hence shape these movements. They are not completely determined by the teachers' actions, though they may be influenced in various ways and for various reasons. Indeed, challenging questions from the learners can force teachers into new insights and into relaxing existing boundaries. Learners equally do force separations and the kinds of connections they want between mathematics and context. They also make decisions about whether they want to stay inside the mathematics or remain on the outside. The power of the notion of complementarity in understanding what learners do is that we have to recognise that there exists, within contexts, within mathematics, and between mathematics and context, numerous disconnections. Therefore, when learners operate in any one of these domains, they are unable to experience and act on the other, and this puts us in better position to explain the difficult challenge for learners to hold two or more in focus at the same time. It is when they are deliberately moved out and across these through challenging questions that they can see or experience their connectedness. This movement is essential in mathemacy. In the project presentations, the vast majority of learners did not connect the graphs and the realities they represented. So although their graphical representations could have been technically correct, their connection to a particular reality was largely invisible and problematic. Challenging questions can be used by both teachers and learners, but their effectiveness is mediated through the relations of power inherent to any teaching and learning setting within the didactical contract. This means that they can be used to negotiate multiple meanings in mathematics and in context, but they can also be used to suppress meaning in classrooms. In this sense, challenging questions can be both controlling and enabling.

Complementarity, mathematics, context and the new South African curriculum

The new mathematics and mathematics literacy curricula in South Africa implore, if not require, teachers to forge connections between mathematics and context. Hence complementarity may continue to be a useful theoretical tool for analysing how teachers develop and enact this curriculum reform imperative. Recent data from reform engaging classrooms extends an understanding of complementarity which demonstrates that this relation of opposition and co-operation is shaped by changes in contexts and in mathematics. That is, the quality and nature of contexts are in a complementarity with the mathematics content and processes chosen by teachers.

This opportunity is provided by data generated from an international Learners' Perspective Study, which focused on three diverse schools in a city in each of twelve countries (Clarke, Keitel and Shimizu, 2006). In South Africa, the study was based in Durban. A range of data was produced including split screen video recordings of teachers, and learners in consecutive grade eight mathematics lessons, post lesson stimulus-recall interviews with learners and teachers and teaching learning materials (Clarke *et al*., 2006). The study was undertaken soon after the implementation of the first Outcomes based Education and 2005 Curriculum reforms (2001-2) in each of three former racially segregated schools (see Sethole, 2005; Goba 2004 for context descriptions[4]), with teachers regarded as competent and well-familiarised with the new curriculum reforms.

For this analysis, the focus is on some of the 14 consecutive lessons delivered in a former 'Indian' school, in which a theme of 'substance abuse' was being pursued. A minute by minute analysis was made of each recorded lesson in terms of four categories of activities – classroom administration and management; whole class teacher-led engagement with mathematical content; whole class teacher-led engagement with context; and learner-led independent work in groups – show the shifts from content to context and vice versa, their duration and domination (see Appendix A). Contrary to some early criticisms of the reform suggesting that a focus on context may be compromising the teaching of mathematical content, sustained classroom observations shows this not to be the case in the hands of a well qualified teacher. The table in Appendix A illustrates engagement with context and content in the first seven lessons while context seems to disappear from lessons 8 to 14. Context and content are used to provide entry into each other but also serve to deny access to each other. Not only are different kinds of mathematics processes and concepts being dealt with across lessons but also a variety of contexts are used. This analysis especially examines more closely the issue of contexts, which have recently come under discussion in mathematics curricula reforms in South Africa (see for e.g. Venkat and Graven 2007; Julie 2006; Julie and Mbekwa, 2005).

[4] Goba's (2004) study focussed on a former 'Indian' school and Sethole (2005) analysed data from an 'African' school and a former 'White' school in the Learners Perspective Study.

An analysis of the kinds of contexts deployed by teachers across the three schools and learners' and teachers' engagement within and across these appear to be shaped by the authenticity or inauthenticity of the context on the one hand; and their distance in space (being near or far from learners' experience) or time (past, present or future) (Sethole 2005, Sethole, Goba, Adler and Vithal, 2006) on the other hand; and maybe characterized as real, realistic, pseudo (Goba 2004) or even imaginary. To illustrate, drawing on Goba's (2004) analysis of lessons 1, 5, 6, 7, and 12, which most directly engaged the context of 'drugs/drug abuse', it may be possible to observe how (in)authencity and nearness/farness of contexts, experienced as real, realistic, pseudo or imaginary facilitated or obstructed teaching and learning of mathematics and context.

Sethole's (2005) categories of authenticity and distance, derives from how context is used in practice – in both texts and classroom interaction – and learners' relation to it. The qualities of authenticity, related to a context's resonance with learners' experience; and distance, featuring the novelty or unfamiliarity of context, together provides a means to see the ways in which the complementarity of mathematics and context is operationalised. Depending on the life experiences of particular learners, an authentic and near context such as the one related to substance abuse, could be described as real because it was present in some learners' lives. In lesson 1 which included questions based on an activity related to reading a magazine article on medicine abuse and another on statistics of drug abuse in the residential area in which the school is located, the teacher barely engaged any mathematics, being concerned about the levels of drug abuse by learners and shaped by his membership of a school committee dealing with this. While the realness of this context seemed to block out the mathematics, it does not assure consistency in learners' interest or engagement with the context. In the post lesson interviews some learners who lived in areas in which drug abuse was rife and with family members involved or affected by it, expressed resentment and sought engagement with mathematics rather than context whilst others argued for greater engagement with context for greater connectedness and meaningfulness of mathematics (Vithal and Gopal, 2005). So while context drew some learners into the mathematics other were repelled and resisted its presence.

By contrast in lesson 12 seeking to teach prime numbers, the context of being a detective to decipher a code in order to infiltrate a gang, which is an imaginary context being both inauthentic and far, is not really referred to or engaged in the lesson. Both teachers and learners focus only on the content and learners barely recognise its relation to the theme though the teacher refers to it as an extension of the theme on drugs.

As the kinds of contexts changed and different aspects of mathematics were dealt with in each lesson, changes in the operationalising of complementarity of mathematics and contexts could be observed. Pseudo contexts, which are inauthentic and far, and typically invented to illustrate some mathematics, were engaged in very limited ways because of the contrived nature of the contexts. This can be seen in lesson 5 which was on developing number patterns or sequences where the context was that of a drug company increasing its sale of drugs. In lesson 6, on packaging of pills to minimize costs, where the context may be described as realistic, being authentic and far in a learner's possible future adult life, a similar engagement with context is observed.

Lesson 7, however, involved a combination of a realistic context on taking pills of different strengths, which then continued into a pseudo context of arranging pills in groups to establish number patterns, and eventually became contextless and abstract focusing on mathematical content of divisibility rules for specified numbers (Goba 2004). The shifts in contexts are related to changes in the mathematics engaged in. The movement between these different forms of context and mathematics is discernible at different points in any one lessons influenced by the different activities and specific items in the worksheets, and the classroom interactions. At times context is made to cooperate with mathematics and used to draw learners into the mathematics and then allowing the context disappear. But how long teachers and learners remain in each and allow or obstruct these border crossings vary.

The categories, however, are not fixed and immutable. Pseudo contexts can be made real when learners bring (or force) their life experiences into the lessons. It is the agency of learners in the complementarity of mathematics and context that must also be considered in a classroom. No doubt the teachers' activity in seeking to honour the didactical contract set up between themselves and learners within a *mathematics* classroom when context is brought into a lesson is ever present and powerful. However the kind and extent of any engagement with mathematics and context is shaped as much by learners and can be accepted or challenged by them.

Concluding Remarks

The caution against the use of context in mathematics curricula derives primarily from a concern that a focus on context may deny particular groups of learners' access into mathematics itself as a self-referential system. This analysis of a complementarity of mathematics and context explains how context may indeed function, through its different forms, to block entry into mathematics, but it also allows the observation that context can co-operate with mathematics. For some learners context is important to generating a pathway into mathematics by making connectedness and relevance of mathematics visible. How context and mathematics obstruct or oppose each other in a mathematics lesson or how they co-operate lie in the choices that teachers and learners make in a classroom in response to a host of different pressures: of curriculum reforms requiring the use of different contexts; and other aspects such as assessment, timetabling, available educational materials; and also their own ideologies, values, attitudes and positions about the teaching and learning of mathematics.

Failure to learn mathematics can therefore be analysed through understanding the mediation of the complementarity of mathematics and context. In constructing learners as 'purveyor of ideology', Mellin Olsen (1987) interprets failure to learn mathematics as failure of school mathematics to provide access to the 'thinking tools of the curriculum'. He explains failure firstly, as learners actively rejecting mathematics, which in turn leads to a conscious resistance; and secondly they get caught in various double binds because they lack the appropriate meta-knowledge of the conflicts inherent in the messages sent to them through lessons and school. How teachers and schools make the conflicts and co-operations between mathematics and context explicit, visible and challengeable can shape and influence how learners choose to participate and traverse the divides. Schools and mathematics classrooms are sites in which multiple ideologies operate – colliding, conflicting and collaborating – and equally are sites for challenging ideologies. The task for mathematics education, especially from a critical perspective is to offer experiences of how to apply the thinking tools of the curriculum in such a way that they are recognised as functional knowledge by learners not only to become critically aware but to transform that awareness into social or political action. The complementarity of context and mathematics shows up the pitfalls and possibilities of such a mathematics pedagogy.

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Appendix A: Table showing 14 consecutive lessons from a former 'Indian' school in the Learners' Perspective Study

	Classroom management and administration: greeting, checking of homework, disciplining of the learners, etc.
	Teacher-led whole class discussion on <u>context</u>
	Teacher-led whole class discussion on <u>mathematics</u>
	Learners working independently on their own on a given task

(Acknowledgement: This table was developed by Busisiwe Goba; for full descriptions of lessons used in this analysis see Goba, 2004)

Mins.	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14
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46														
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50														
52														
54										End				
56														
58			End											
60									End					
62		End			End	End	End				End		End	End
64				End										
66														
68														
70														
72														
74														
76														
78														
80														
82														
84														

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